國立臺灣大學 109 學年度碩士班招生考試試題

1. (15%) Find the extrema of the function $f(x,y,z) = xyz^{1/3}$ on the region $x+y+z = 1$, $x \geq -1, y \geq -2, z \geq -3$.

2. (10%) Suppose $f: R \to R$ is differentiable and satisfies $f'(x) > f(x)$ for all real x. Show that if $f(0) = 0$ then $f(x) > 0$ for all $x > 0$. In your proof, state clearly on theorems you used.

3. (15%) (a) (8%) Evaluate the following integral

$$\int_{-\infty}^{\infty} \frac{x - \sin x}{x^3} dx$$

and (b) (7%) provide an argument to verify that the existence of this integral.

4. (15%) For a real symmetric positive definite $p \times p$ matrix A and a vector $\mathbf{v} \in R^p$, show that

$$\int_{R^p} \exp\left(-\mathbf{x}^T A\mathbf{x} + 2\mathbf{v}^T\mathbf{x}\right) d\mathbf{x} = \pi p/2\sqrt{\det A}\exp(\mathbf{v}^T A^{-1}\mathbf{v}).$$

You may assume that $\exp(A)$ is a well-defined matrix.

5. (15%) Prove that the initial value problem $\dfrac{dx}{dt} = 3x + 85\cos x,\ \ x(0) = 77$, has a solution $x(t)$ defined for all $t \in R$.

6. (15%) Suppose that $f(x)$ is defined on [-1,1], and that $f^{(3)}(x)$ is continuous. Show that the series $\sum_{i=1}^{\infty}\left[n\left(f(1/n) - f(-1/n)\right) - 2f'(0)\right]$ converges.

7. (15%) Use Stokes' Theorem to evaluate

$$\iint_S \operatorname{curl}\vec{F} \cdot d\vec{S}.$$ Here $\vec{F} = y\vec{i} - x\vec{j} + yx^3\vec{k}$ and S is the portion of the sphere of radius 4 with $z \geq 0$ and the upwards orientation.

試題隨卷繳回