Statistics

1. (20 pts) Define or state the following terms:
(1a) random variable. (1b) probability function. (1c) convergence in distribution. (1d) uniformly most powerful test.

2. (20 pts) Let $X_1, \ldots, X_n$ be a random sample drawn from a finite population $\{w_1, \ldots, w_N\}$. Find an unbiased estimator of the population variance $\sigma^2 = \sum_{i=1}^N (w_i - \mu)^2/N$, where $\mu = \sum_{i=1}^N w_i/N$.

3. (20 pts) Let X be a continuous random variable with the cumulative distribution function $F(x)$ and $X_{(1)}, \ldots, X_{(n)}$ be the corresponding order statistics of a random sample $X_1, \ldots, X_n$. Give the joint distribution of $F(X_{(i)})$, $F(X_{(j)})$, and $F(X_{(k)})$ for any i, j, and k with $1 \leq i < j < k \leq n$.

4.(20 pts) Let $X_1, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance 1. Find the uniformly minimum variance unbiased estimator of μ^2.

5. Let $X_1, \ldots, X_n$ be a random sample from a geometric distribution $P(X = x) = \theta(1 - \theta)^{x-1} I_{\{1,2,\ldots\}}(x)$ and θ have a uniform prior distribution on $(0, 1)$.
(5a) (10 pts) Give the posterior distribution of θ.
(5b) (10 pts) Find the Bayes estimator of θ under the square error loss $L(\theta, \delta(X_1, \ldots, X_n)) = (\delta(X_1, \ldots, X_n) - \theta)^2$

試題隨卷繳回