Problem 1. (25%)

The classical fluidic mechanics example about an incompressible Newtonian fluid flowing through a gap (height = $2h$) between two horizontal, infinite fixed parallel plates is illustrated in *Figure 1*.

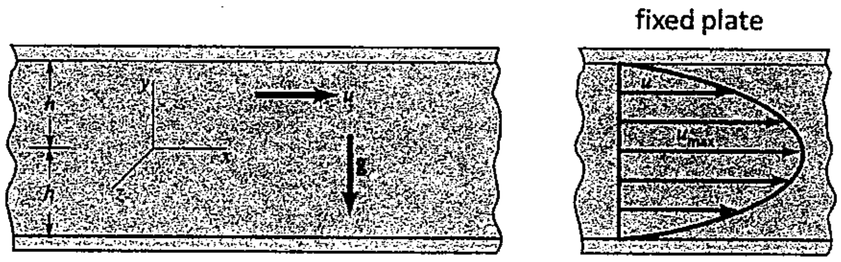

Figure 1

By solving the following set of the Navier-Stokes equations with proper initial and boundary conditions, the fluid velocity profiles along x, y, and z directions (u, v, and w) can be derived, respectively.

(x direction)

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

(y direction)

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

(z direction)

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

(a) Show the fully developed laminar flow velocity profile u (y) as below. (5%)

$$u=\frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2-h^2)$$

(b) Assume that the gap height is adjustable. As far as the viscosity measurement is concerned, show that the viscosity of the fluid measured μ would be proportional to h^3 while the volume flow rate q and pressure drop $\Delta p/L$ are kept constant. (5%)

(c) Show how the shear stress at the plate surface τ_w is dependent on the fluid viscosity μ. (5%)

(d) It is known that the Fanning friction factor f is defined as the following equation, where V is the average velocity of the fluid. Try your best to correlate f to Re for the fluid under a laminar flow condition. (5%)

$$f\equiv\frac{\tau_w}{\rho V^2/2}$$

(e) Explain the general physical significance of f for the fluid friction in the flow channel from the aspect of the Bernoulli equation. (5%)

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Problem 2. (25%)

Consider transient 1-D heat conduction in a semi-infinite metallic solid depending under different surface conditions, as illustrated in *Figure 2*: (i) kept at a constant surface temperature T_s, (ii) kept at a constant heat flux q_o'', and (iii) exposed to a fluid with a temperature of T_∞ and a convective coefficient h.

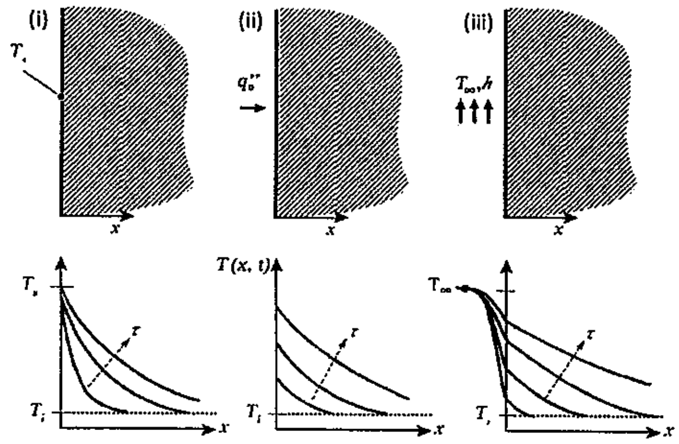

Figure 2

(a) Formulate the governing partial differential equation for solving *T(x, t)* of the metallic solid. (4%)

(b) Write down the boundary conditions at surface $x = 0$ for all time for case (i), (ii), and (iii), respectively. (6%)

(c) Solve *T(x,t)* for the metallic solid in case (i) with a homogeneous initial temperature T_i ($< T_s$). (9%)

(d) Comment in which case the Biot (*Bi*) number is important and explain why. (3%)

(e) Derive the overall heat transfer coefficient *U* for case (iii) when the fluid and metallic solid have finite thickness of d_f and d_m, respectively. (3%)

Problem 3. (25%)

Consider a chemical reactor containing a binary fluid mixture of solute species A and solvent species B initially. The molar flux of A, $\mathbf{W}_A$, is the result of two contributions: $\mathbf{J}_A$, the molecular diffusion flux relative to the bulk motion of the fluid produced by a concentration gradient, and $\mathbf{B}_A$, the flux resulting from the bulk motion of the fluid. (PS: The bold-faced symbols represent vectors.)

$$\mathbf{W}_A = \mathbf{J}_A + \mathbf{B}_A$$

(a) Justify the following equation. (5%)

$$\mathbf{W}_A = -D_{AB}\nabla C_A + C_A\mathbf{V} = -cD_{AB}\nabla y_A + y_A(\mathbf{W}_A + \mathbf{W}_B)$$

(b) List THREE distinct experimental conditions that satisfy the following equation. (5%)

$$\mathbf{W}_A = \mathbf{J}_A$$

(c) Derive the following equation by applying the mole balance to species A, which flows and reacts in an element of volume. (5%)

$$-\frac{\partial W_{Ax}}{\partial x} - \frac{\partial W_{Ay}}{\partial y} - \frac{\partial W_{Az}}{\partial z} + r_A = \frac{\partial C_A}{\partial t}$$

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Problem 3. (Cont'd)

(d) Prove that the one-dimension, steady-state concentration profile of C_A in the reactor can be modeled by the following ODE when both diffusion and convective transport are important. Start your proof from the equation in (c). (5%)

$$D_{AB}\frac{d^2C_A}{dz^2} - U_z\frac{dC_A}{dz} + r_A = 0$$

(e) Consider a homogeneous first order reaction A → P takes place in the reactor with the rate law of $r_A = -kC_A$. Solve $C_A(z)$ under the circumstance of $D_{AB} << U_z$ (the fluid velocity along z direction). (5%)

Problem 4. (25%)

The heat transfer correlation relating the Nusselt number (Nu) to the Reynolds (Re) and Prandtl (Pr) numbers for flow around a sphere is

$$Nu = 2 + 0.6Re^{1/2}Pr^{1/3}$$

The equation can be applied to single spherical catalyst pellets (with a diameter, d_p) in a packed bed reactor (PBR) passing a plug flow of the reactant A at a uniform fluid velocity U and with a constant fluid density ρ. The Nu, Re, and Pr for the spherical pellets are defined as follows.

$$Nu = \frac{hd_p}{k_t} \qquad Re = \frac{U\rho d_p}{\mu} \qquad Pr = \frac{\mu C_p}{k_t}$$

(a) Explain (i) each term in Nu and Pr definitions and (ii) the physical meanings of Nu and Pr. (6%)

(b) Comment how the heat flux q transferred from the bulk fluid to the pellet surface is varied from a very low Re number to a high Re number (while the boundary layer remains laminar). (4%)

(c) By analogy, the correlation for mass transfer to flow around a spherical pellet can be described as follows.

$$Sh = 2 + 0.6Re^{1/2}Sc^{1/3}$$

where

$$Sh = \frac{k_c d_p}{D_{AB}} \qquad Sc = \frac{v}{D_{AB}}$$

Explain the physical meanings of Sh and Sc. (5%)

(d) Calculate the mass flux of reactant A, W_{Ar}, to a single catalyst pellet 1 cm in diameter suspended in a large body of liquid. The reactant is present in dilute concentrations, and the reaction is considered to take place instantaneously at the external pellet surface (*i.e.*, $C_{As} \approx 0$). The bulk concentration of the reactant is 1.0 M, and the fluid velocity U is 0.1 m/s. The kinetic viscosity is 0.5 centistoke (1 cS = 10^{-6} m²/s), and the liquid diffusivity of A is 10^{-10} m²/s. T = 300 K. Hint: $W_{Ar} = k_c(C_{Ab} - C_{As})$ (10%)