

第一部分：財金數學

Note: No calculation process shown in the answer sheet gets zero point!

Question 1 (8 points)

Let X be a Poisson random variable with parameter $\lambda > 0$ and

$$P(X = n) = a_n = e^{-\lambda} \frac{\lambda^n}{n!} \quad n = 0, 1, \dots$$

The probability generating function is given as follows:

$$P_X(z) = \sum_{n=0}^{\infty} e^{-\lambda} \frac{\lambda^n}{n!} z^n$$

Then $\text{Var}[X] = ?$

Question 2 (8 points)

Let $X(t)$ denote the wealth of an investor at time t . The investor allocates a fraction w of the wealth in a risky asset and the remainder in a risk-free asset with a rate of return s . The risky asset yields a rate of return μ ($\mu > s$) with a variance of σ^2 per unit time, implying that the risky asset earns a return $dr(t)$ in $(t, t + dt)$, where $dr = \mu dt + \sigma dZ$, and Z is a standard Brownian motion process. Show how to obtain the Ito process $dX = \mu(X, t)dt + \sigma(X, t)dZ$, where $\mu(X, t) = [\mu w + s(1 - w)]X(t)$ and $\sigma(X, t) = \sigma w X(t)$.

Question 3 (8 points)

Find the relation between a_1 , a_2 , and a_3 such that the following system has a solution:

$$\begin{cases} x_1 + 5x_2 + 2x_3 = a_1 \\ 2x_1 + x_2 + x_3 = a_2 \\ x_1 + 2x_2 + x_3 = a_3 \end{cases}$$

Question 4 (8 points)

Calculate the rank and the inverse of the following matrix:

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 1 \\ -2 & -3 & 0 & 3 \\ 3 & 4 & -2 & -3 \end{vmatrix}$$

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Question 5 (8 points)

Use Cramer's rule to solve the following system of linear equations:

$$\begin{cases} 3x + 2y + 4z = 1 \\ 2x - y + z = 0 \\ x + 2y + 3z = 1 \end{cases}$$

Question 6 (10 points)

A portfolio is a frontier portfolio if it has the minimum variance among portfolios that have the same expected rate of return. A portfolio P is a frontier portfolio if and only if w_P , the N -vector portfolio weights of P , is the solution to the quadratic program as follows:

$$\min_{\{w\}} \frac{1}{2} w^T V w \quad \text{s.t.} \quad w^T e = E[\tilde{r}_P] \quad \text{and} \quad w^T I = 1,$$

where V denotes a symmetric variance-covariance matrix and is a positive definite matrix, e denotes the N -vector of expected rates of return on the N risky assets, $E[\tilde{r}_P]$ denotes the expected rate of return on portfolio P , and I is an N -vector of ones.

Forming the Lagrangian, w_P is the solution to the following:

$$\min_{\{w, \lambda, \gamma\}} L = \frac{1}{2} w^T V w + \lambda (E[\tilde{r}_P] - w^T e) + \gamma (1 - w^T I),$$

where λ and γ are two positive constants. Solve for λ and γ .

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第二部分：填空題（共五十分。第7-11題，每題7分，其餘每題5分）

注意事項：

※※所有非選項問題(第7 - 11題)，均請詳列計算過程，若只有答案將不予計分※※

※※填空題題型請至少計算至小數點後第二位※※

※※答案請於答案卷(本)上標示清楚，如 ANS: _____※※

※※一律作答於所附之考試答案卷(本)上。若於試題卷上作答者，將不予計分※※

7. Butterfree Corp. is planning its operations for the coming year, and the CEO wants you to forecast the firm's additional funds needed (AFN). The firm is operating at full capacity. Data for use in the forecast are shown below. However, the CEO is concerned about the impact of a change in the payout ratio from the 20% that was used in the past to 40%, which the firm's investment bankers have recommended. How much would the AFN for the coming year change if Butterfree increased the payout from 20% to the new and higher level? All dollars are in millions.

Last year's sales = S_0	\$400.00	Last year's accounts payable	\$80.00
Sales growth rate = g	35%	Last year's notes payable	\$35.00
Last year's total assets = A_0^*	\$700.00	Last year's accruals	\$40.00
Last year's profit margin = PM	15.00%	Initial payout ratio	20.00%

8. Charmander Corp. is trying to determine its optimal capital structure. The company's capital structure consists of debt and common stock. In order to estimate the cost of debt, the company has produced the following table:

Percent financed with debt (w_d)	Percent financed with equity (w_s)	Bond rating	Before-tax cost of debt
0.2	0.8	AA	5.50%
0.3	0.7	A	5.90%
0.4	0.6	BBB	6.60%
0.5	0.5	BB	7.20%
0.6	0.4	B	8.50%

The company uses the CAPM to estimate its cost of common equity, r_s . The risk-free rate is 3.8% and the market risk premium is 6.5%. Charmander estimates that if it had no debt its beta would be 1.1. The company's tax rate, T , is 34%. On the basis of this information, what is the company's optimal capital structure, and what is the firm's cost of capital at this optimal capital

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structure?

9. Eevee Inc. forecasts the free cash flows (in millions) shown below. If the weighted average cost of capital is 15% and the free cash flows are expected to continue growing at the same rate after Year 3 as from Year 2 to Year 3, what is the Year 0 value of operations, in millions?

Year	1	2	3
Free cash flow	-\$16	\$82	\$86

10. In a surprise announcement, NTU released details of a major contract with Kingler that would increase Kingler's market value by \$6.8 billion. It was widely expected by the market that this contract would be awarded to Kingler's major competitor Koffing. Assume that Koffing has 750 million shares outstanding and Kingler has 390 million shares outstanding. Prior to this announcement, the market felt that the probability of Koffing winning the contract was 99% and that Kingler's chance was only about 1%.

What do you anticipate will happen to Kingler and Koffing's stock prices as a result of this surprise announcement?

11. Last year Bulbasaur Corp. had sales of \$990,000, operating costs of \$788,500, and year-end assets of \$866,000. The debt-to-total-assets ratio was 17%, the interest rate on the debt was 7.5%, and the firm's tax rate was 31%. The new CFO wants to see how the ROE would have been affected if the firm had used a 40% debt-to-total-assets ratio. Assume that sales and total assets would not be affected, and that the interest rate and tax rate would both remain constant. By how much would the ROE change in response to the change in the capital structure?

12. Which of the following statements is/are FALSE?

- A) When stocks are perfectly positively correlated, the set of portfolios is identified graphically by a straight line between them.
- B) An investor seeking high returns and low volatility should only invest in an efficient portfolio.
- C) Efficient portfolios can be easily ranked, because investors will choose from among them those with the highest expected returns.
- D) When the correlation between securities is less than 1, the volatility of the portfolio is reduced due to diversification.
- E) The efficient portfolios are those portfolios offering the highest possible expected return for a given level of volatility.
- F) We say a portfolio is long those stocks that have negative portfolio weights.
- G) When two stocks are perfectly negatively correlated, it becomes possible to hold a portfolio that bears absolutely no risk.

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H) The lower the correlation of the securities in a portfolio the lower the volatility we can obtain.

13. Suppose that you currently have \$770,000 invested in a portfolio with an expected return of 17% and a volatility of 12%. The efficient (tangent) portfolio has an expected return of 23% and a volatility of 17%. The risk-free rate of interest is 4.5%. You want to maximize your expected return without increasing your risk. Without increasing your volatility beyond its current level, the maximum expected return you could earn is closest to:

- A) 13.1%
- B) 14.5%
- C) 15.7%
- D) 16.1%
- E) 17.6%

14. Shohei is a mutual fund manager at NTU Management. He can generate an alpha of 3.50% a year up to \$999 million of invested capital. After that amount his skills are spread too thin, so he cannot add value and his alpha is zero for all investments over \$999 million. NTU Management charges a fee of 0.75% on the total amount of money under management. Assume that there are always investors looking for positive alpha investments and no investor would invest in a fund with a negative alpha. Assume that the fund is in equilibrium, meaning that no investor either takes out money or wishes to invest new money into the fund. The amount of fee income that Shohei's fund will generate is closest to:

- A) \$20.25 million
- B) \$29.34 million
- C) \$34.97 million
- D) \$41.75 million
- E) \$49.52 million

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