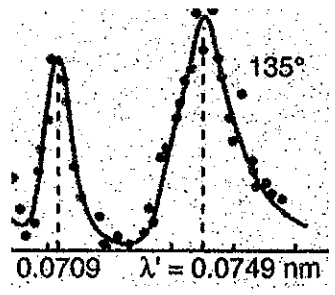
1. The back scattered X-ray spectrum at 135° from a Compton scattering experiment has two peaks as shown on the right figure. The incident X-rays have wave length ~0.0709 nm. Derive the formula of wave length shift at the scattering angle θ for a photon elastically scattering off an initially stationary charged particle (with mass m) to explain the observed wave lengths of these two peaks. You need to plug in the numerical values in order to match the two peak values from the derived formula. Note that $h/(m_e c)$=0.0024 nm, where h, m_e, and c denote the Planck's constant, mass of electron, and speed of light in vacuum, respectively. (20 points)



2. Find the ground state energy and wave function for a particle with mass m trapped in a delta-function potential, $V(x)= -\alpha\delta(x)$, where α is a positive constant. (20 points)

3. If a single particle (with mass m) system has the following Hamiltonian (H) and Q matrix representations, one measure its Q value to be q at $t = 0$.

$$H = \hbar\omega\begin{pmatrix}2 & 0 & 0\\ 0 & 0 & -i\\ 0 & i & 0\end{pmatrix}, \qquad Q = q\begin{pmatrix}1 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 5\end{pmatrix}, \ \Psi(t=0) = \begin{pmatrix}1\\ 0\\ 0\end{pmatrix}.$$

Find the eigen values and eigen vectors of H. What is $\Psi(t)$ and the probability to have particle energy $\hbar\omega$ at time t, $t > 0$? (20 points)

4. Two non-interacting identical fermions trapped in an infinite potential well with $0 \le x \le a$. Write down the ground state and first excited state wave functions for this two-fermion system. You need to find the single particle wave function $\Psi(x,t)$ first and use $\chi_\uparrow$ ($\chi_\downarrow$) to denote the spin-up (spin-down) state. You should also use proper index, e.g. x_1 and χ_1, to specify particle 1 state, and write down all degenerate states. (20 points)

5. The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science". Describe briefly the mentioned works/meanings in quantum physics from the above statement. (20 points)

試題隨卷繳回