請於答案卷上作答，於試題卷上作答者，不予計分。

1. **Figure A** shows a simple pendulum, where all the mass is concentrated at the end point. (a) Write the equations of motion for the pendulum.【計分：3 分】(b) By assuming the motion is small enough that $\sin\theta \cong \theta$, find the transfer function $\Theta(s)/T_c(s)$.【計分：2 分】(c) For the nonlinear equations of motion considered in part (a), rewrite the equation of motion in state-variable form, with $\mathbf{x} = [x_1 \quad x_2]^T = [\theta \quad \dot{\theta}]^T$.【計分：4 分】(d) Suppose that the (normalized) applied torque has a nominal value of $u_0 = 0$. Derive the equilibrium points for the system and determine the corresponding small-signal linear models.【計分：6 分】(e) Linearization by feedback can be accomplished by subtracting the nonlinear terms out of the equations of motion and adding them to the control. The result will be a linear system, provided that the computer implementing the control has enough capability to compute the nonlinear terms fast enough and the resulting control does not cause the actuator to saturate. Linearize the system by using nonlinear feedback.【計分：5 分】

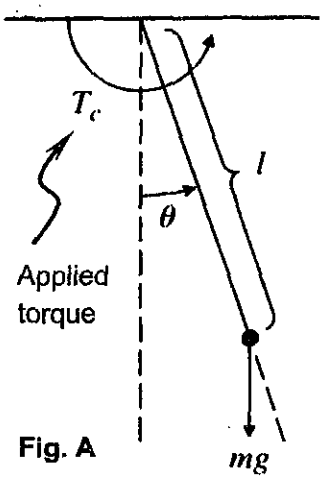

Fig. A

2. Consider the continuous rolling mill depicted in **Fig. B**. Suppose that the motion of the adjustable roller has a damping coefficient b, and that the force exerted by the rolled material on the adjustable roller is proportional to the material's change in thickness: $F_s = c(T - x)$. Suppose further that the DC motor has a torque constant K_t and a back-emf constant K_e, and that the rack-and-pinion has effective radius of R. (a) What are the inputs to this system? The output?【計分：4 分】(b) Without neglecting the effects of gravity on the adjustable roller, draw a block diagram of the system that explicitly shows the following quantities: $V_s(s)$, $I_a(s)$; $F(s)$ (the force the motor exerts on the adjustable roller), and $X(s)$.【計分：8 分】(c) Simplify your block diagram as much as possible while still identifying output and each input separately.【計分：3 分】

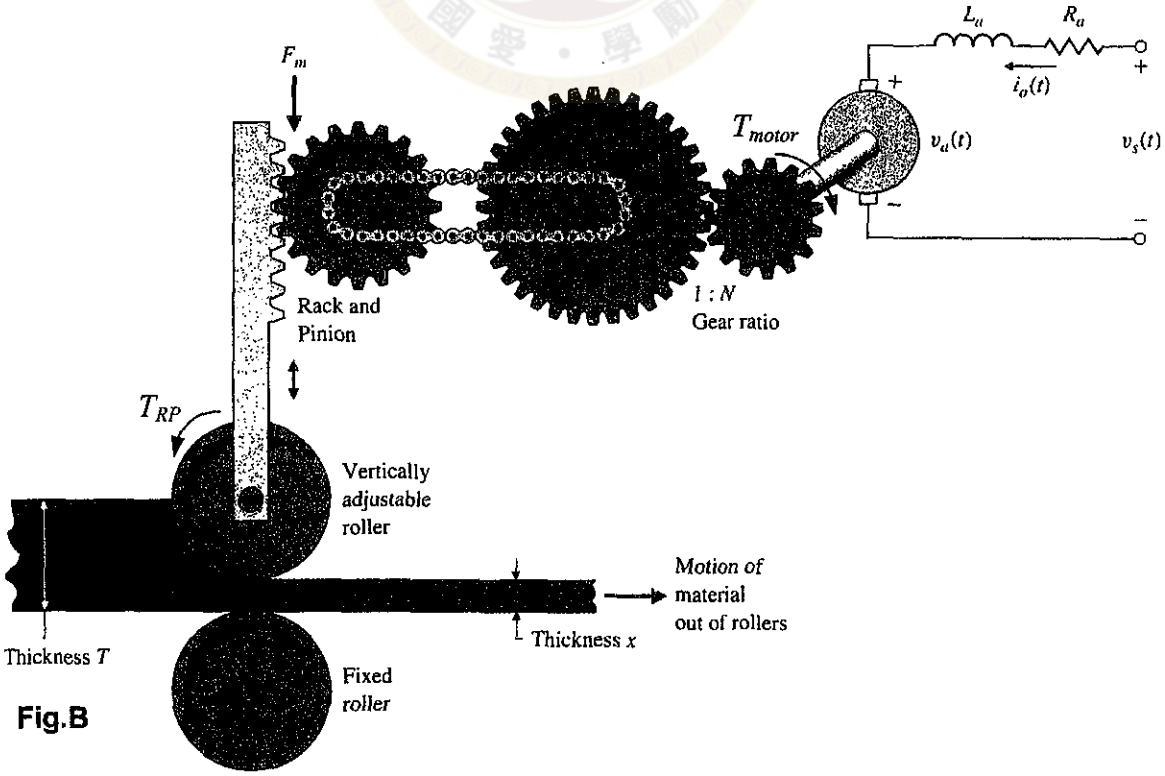

Fig.B

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3. Suppose you are given the system depicted in **Fig. C-(a)**, where the plant parameter a is subject to variations. (a) Find $G(s)$ so that the system shown in **Fig. C-(b)** has the same transfer function from r to y as the system in **Fig. C-(a)**.【計分：5 分】(b) Assume that $a = 1$ is the nominal value of the plant parameter. What is the system type and the error constant in this case? 【計分：10 分】(c) Now assume that $a = 1 + \delta a$, where δa is some perturbation to the plant parameter. What is the system type and the error constant for the perturbed system?【計分：10 分】

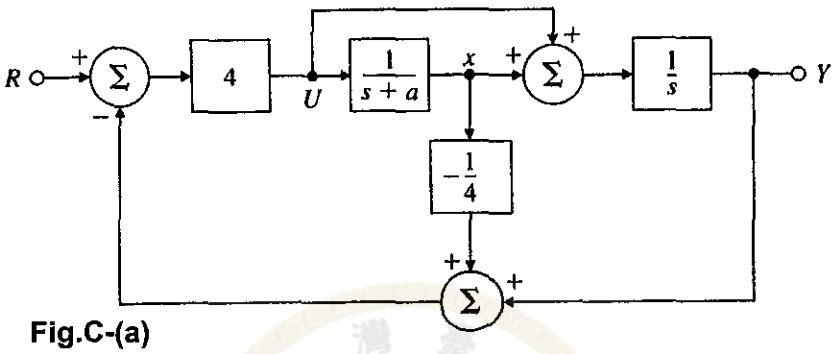

Fig.C-(a)

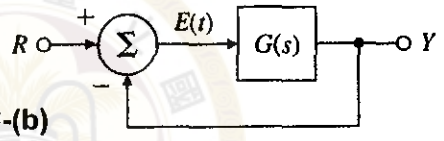

Fig.C-(b)

4. Suppose you are given a system with the transfer function $L(s) = (s + z)/[(s + p)^2]$, where z and p are real and $z > p$. Show that the root-locus for $1 + KL(s) = 0$ with respect to K is a circle centered at z with radius given by $r = (z - p)$. **< Hint >:** Assume $s + z = re^{j\phi}$ and show that $L(s)$ is real and negative for real ϕ under this assumption.【計分：10 分】

5. Consider the system $\frac{d\mathbf{x}(t)}{dt} = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix}u(t)$, $y(t) = [1 \quad 2]\mathbf{x}(t)$. (a) Determine the state controllability and observability of the system.【計分：8 分】 (b) Let $u(t) = -\mathbf{K}\mathbf{x}(t)$, where $\mathbf{K} = [k_1 \quad k_2]$, and k_1 and k_2 are real constants. Determine if and how controllability and observability of the closed-loop system are affected by the elements of $\mathbf{K}$.【計分：12 分】

6. (a) What Bode plot characteristic is the best indicator of the closed-loop step response overshoot?【計分：5 分】
(b) What Bode plot characteristic is the best indicator of the closed-loop step response rise time?【計分：5 分】

試題隨卷繳回