

1. Given a matrix $A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$.
- (a) (15%) If $S^{-1}AS = \Lambda$ where S is a nonsingular matrix and Λ is a diagonal matrix, find S and Λ .
- (b) (15%) Find the Q-R factorization of A .
2. (20%) Consider a scalar function $\Phi(r, \theta, \phi)$ in a spherical coordinates, where $z = r \cos \theta$ is the usual z component in the rectangular coordinates. Given that $\nabla \Phi = 2r(\cos \phi)(\sin \theta)\mathbf{i}_r + r(\cos \phi)(\cos \theta)\mathbf{i}_\theta - r(\sin \phi)\mathbf{i}_\phi$, find the Laplacian of Φ .
3. $y = y(x)$ is a function of the independent variable x . Solve the following differential equations.
- (a) (10%) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - 3)y = 0$.
- (b) (15%) $x^2 \frac{dy}{dx} + 2y + x = 0$.
- (c) (25%) $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = 6xe^x + 2x \cos x$, $\left. \frac{dy}{dx} \right|_{x=0} = 0$, $y(0) = 0$.