第 1~13 複選題（答案可能有一個或多個，完全答對才給分，答錯不倒扣），考生應作答於「答案卡」；
第 14~15 計算題，考生需寫出計算過程及答案。

1. (10%) Here is an augmented matrix in which * denotes an arbitrary number and # denotes a nonzero number.

$$\begin{bmatrix} \# & * & * & * & * & * \\ 0 & \# & * & * & 0 & * \\ 0 & 0 & \# & * & * & \# \\ 0 & 0 & 0 & 0 & \# & * \end{bmatrix}$$

Which of the following statements is (are) true?
(A) The given augmented matrix is consistent and the solution is unique.
(B) The given augmented matrix is inconsistent and the solution is unique.
(C) The given augmented matrix is consistent and the solution is not unique.
(D) The given augmented matrix is inconsistent and the solution is not unique.
(E) None of the above.

2. (5%) Assume A is an $m \times n$ matrix. If rank $A = n$, then
(A) The columns of A are linearly independent.
(B) $A\mathbf{x} = \mathbf{b}$ has at least one solution for every $\mathbf{b}$ in R^m.
(C) Every column of its reduced row echelon form contains a pivot position.
(D) $A\mathbf{x} = \mathbf{b}$ has at most one solution for every $\mathbf{b}$ in R^m.
(E) None of the above statements is true.

3. (5%) For the four vectors below, which of the following statements is (are) true?

$$\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 8 \\ 0 \\ 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

(A) They are linearly independent.
(B) They span R^3.
(C) They are linearly dependent.
(D) They do not span R^3.
(E) None of the above.

4. (5%) Let V be a subspace of R^n with dimension k. Which of the following statements is (are) true?
(A) Every linearly independent subset of V contains at least k vectors.
(B) Any finite subset of V containing more than k vectors is linearly dependent.
(C) $n \geq k$.
(D) Any finite subset of V containing less than k vectors is linearly independent.
(E) None of the above.

5. (5%) Suppose that s, t, and u are vectors in R^n such that s is orthogonal to u and u is orthogonal to t. Then
(A) s is orthogonal to t
(B) For any orthogonal $n \times n$ matrix P, we have that Pu is orthogonal to both s nd t
(C) For any orthogonal $n \times n$ matrix P, we have that Ps is orthogonal to u

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(D) $s + t$ is orthogonal to u

(E) None of the preceding statements are true.

6. (5%) Let M be an inner product space

(A) Since the inner product is an integral of a product of functions, M is a function space

(B) $c\langle u, v\rangle = \langle cu, cv\rangle$ for all vectors u and v in M and for every scalar c

(C) $\langle T(u), v\rangle = \langle u, T(v)\rangle$ for all vectors u and v in M and for every linear operator T on M

(D) If M is finite-dimensional, then M contains an orthonormal basis

(E) None of the preceding statements are true.

7. (5%) An $m \times n$ matrix P is invertible if

(A) The reduced row echelon form of P is I_n

(B) The columns of P are linearly independent

(C) The columns of P span R^m

(D) The rows of P are linearly independent

(E) None of the preceding statements is true.

8. (5%) Suppose that S is an arbitrary $n \times n$ matrix. Then

(A) det S is the sum of its diagonal entries

(B) det $S^2 = 2$ det S

(C) det S is a vector in R^n

(D) det S = det S^T

(E) None of the preceding statements is true.

9. (5%) Let A be a subset of R^3 containing two or more vectors. Then:

(A) The span of any two vectors in A is a plane in R^3

(B) If A contains more than three vectors, then A is linearly independent

(C) The span of any two nonzero vectors in A is a plane in R^3

(D) Every vector in A is in the span of A

(E) None of the preceding statements is true.

10. (8%) The solution for $f(t)$ of the following equation

$$f(t) = 3t^2 - e^{-t} - \int_0^t f(\rho) e^{t-\rho} d\rho$$

has the following form:

$f(t) = g(t) - k_1 e^{-k_2 t}$, where k_1~k_2 are constants and $g(t)$ is a function containing only polynomial terms like $k_n t^n$.

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Which of the following item(s) is (are) true: (A). $k_1+k_2=3$ (B). $k_2+g(0)=1$ (C). $g(1)+g(0)=4$ (D). $g(1)+g(-1)=8$ (E) none of above.

11. (7%) The function f(x)=|x| can be expressed as a Fourier series on the interval $-\pi \le x \le \pi$, where

$$f(x)=|x|=a_0+\sum_{n=1}^{\infty}\left[a_n\cos(nx)+b_n\sin(nx)\right]$$

Which of the following item(s) is(are) true: (A) $a_0=\pi$ (B) $a_2+b_3=0$ (C) $b_2+b_4=0$ (D) $a_1+a_3=-\frac{-4\pi}{9}$ (E) none of above.

12.~13. We are going to solve of the following differential equation system:

$$\frac{dx_1}{dt}=3x_1+x_2-x_3$$
$$\frac{dx_2}{dt}=x_1+3x_2-x_3\ ,$$
$$\frac{dx_3}{dt}=3x_1+3x_2-x_3$$

with initial values as $x_1(0)=4$, $x_2(0)=7$ and $x_3(0)=7$. The solution has the form as

$$x_1=A_{11}e^{\lambda_1 t}+A_{12}e^{\lambda_2 t}+A_{13}e^{\lambda_3 t}$$
$$x_2=A_{21}e^{\lambda_1 t}+A_{22}e^{\lambda_2 t}+A_{23}e^{\lambda_3 t}\ ,$$
$$x_3=A_{31}e^{\lambda_1 t}+A_{32}e^{\lambda_2 t}+A_{33}e^{\lambda_3 t}$$

where $|\lambda_1|\le|\lambda_2|\le|\lambda_3|$ and A_{ij}s are constants. Which of the following item(s) is(are) true:

12. (5%) (A) $\lambda_1+\lambda_2=3$ (B) $2\lambda_1+\lambda_3=3$ (C) $\lambda_2+\lambda_3=3$ (D) $\lambda_1+\lambda_3=3$ (E) none of above.

13. (5%) (A). $x_1(1)+2x_2(1)=6e$ (B). $x_1(1)+x_3(1)=2\cos(1)$ (C). $x_2(1)+x_3(1)=8e$ (D). $x_1(1)-x_2(1)=2\cos(1)$ (E) none of above.

14. (19%) The initial value problem

$$\frac{1}{5}\frac{d^2x}{dt^2}+\frac{6}{5}\frac{dx}{dt}+2x=5\cos(4t),\qquad x(0)=\frac{1}{2},\qquad x'(0)=0$$

has the solution with the form:

$$x(t)=\frac{1}{102}\left\{f(t)\left[k_1\cos(t)+k_2\sin(t)\right]+k_3\cos(k_4t)+k_5\sin(k_4t)\right\}$$

k_1, k_2, k_3, k_4 and k_5 are constants. $f(t)$ is a function of t. Find k_1, k_2, k_3, k_4, k_5 and $f(t)$.

15. (6%) The differential equation

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\ln(x)$$

has the solution with the form:

$$y(x)=c_1x+c_2g_1(x)+p_1+g_2(x)$$

c_1 and c_2 are arbitrary constants. p_1 is a constant. $g_1(x)$ and $g_2(x)$ are functions of x. Find p_1 and $g_1(x)$.

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