※ 注意：請於試卷上依序作答，並應註明作答之部份及其題號。

I. Do problems 1 - 4 by filling in the blanks:

1. (15%) The yellow light of sodium vapor lamps frequently employed in highway illumination is a spectral line arising from the $3p$ to $3s$ transitions in ^{11}Na.

   (a) Given that $E_{3p} - E_{3s} = 2.1\,\text{eV}$, what is the wavelength of this line? ________

   (b) The line is split by the spin-orbit interaction into two components. Given that the spin-orbit interaction splits the $3p$ level by an energy $\Delta E = 2.1 \times 10^{-3}\,\text{eV}$, evaluate the separation in wavelength of the two components of the line. ________
   Note that you are not told by how much the $3s$ level is split by the spin-orbit interaction.

   (c) Specify the selection rules under which the transitions involved in emission of the two components of the line are allowed. ________

2. (10%) Consider two electrons in a spin singlet state.

   (a) If a measurement of the spin of one of the electrons shows that it is in a state with $s_z = \hbar/2$, what is the probability that a measurement of the z-component of the spin of the other electron yields $s_z = \hbar/2$? ________

   (b) If a measurement of the spin of one of the electrons shows that it is in a state with $s_y = \hbar/2$, what is the probability that a measurement of the x-component of the spin yields $s_x = -\hbar/2$ for the second electron? ________

3. (10%) Consider the capture of a π^- by a deuteron (which is the nucleus of deuterium atom and is denoted as d). A slow pion in liquid deuterium loses energy by a variety of mechanisms, until it finally ends up in one of the Bohr orbits about (pn) nucleus, and is captured through the action of the nuclear forces by the reaction

$$\pi^- + d \rightarrow n + n.$$

The pion has spin 0 and negative parity whereas the deuteron has spin 1 and positive parity. Also, the parity of a two-particle orbital state of angular momentum l has parity $(-1)^l$. Note that, in the reaction above, the total angular momentum, charge, and parity are conserved.

   (a) Assume that the pion is captured from the lowest Bohr orbit. In terms of the spectroscopic notatin $^{2S+1}L_J$, the state of the two neutrons must be ________.

   (b) Assume that the pion is captured from a P orbit. In terms of the spectroscopic notatin $^{2S+1}L_J$, the state of the two neutrons must be ________.

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4. (15%) Consider an x-ray beam with wavelength $\lambda = 1.00$ Å incident on free electrons. If the radiation scattered from free electrons is viewed at 60° to the incident beam:
   (a) What is the Compton wavelength shift of the x-ray? ________
   (b) What kinetic energy is given to a recoiling electron? ________
   (c) What percentage of the incident photon energy is lost in the collision? ________

II. Do problems 5 - 7 by giving relevant calculation details:

5. A particle with mass m is in an even mixture of the ground state and the 1st excited state in a one dimensional infinite potential well with width a. Determine its probability density function $P(x,t) = \Psi^*(x,t)\Psi(x,t)$, where $\Psi(x,t)$ is the corresponding wave function (5%). What is the expected energy and energy uncertainty of this particle (5%)? How about the expected particle momentum, $p(t)$, and related uncertainty (10%)?

6. There are many experiments played an important role in the development of modern physics. Describe the set-up and experimental findings for the following ones: (a) Frank-Hertz experiment (5%), (b) Michelson-Morley experiment (5%), and (c) Stern-Gerlach experiment (5%).

7. Neutrino mixing can be explained by a simple theoretical model that their mass eigenstates and flavor states are not coincides. So a well defined flavor state produced at time 0 along with a charged lepton in weak interactions should be described by a superposition of different mass states. Assuming a special case that only two flavors needs to be considered and the mixing angle is θ, the following relations will hold: $\nu_\alpha = \nu_1 \cos\theta - \nu_2 \sin\theta$ and $\nu_\beta = \nu_1 \sin\theta + \nu_2 \cos\theta$, where ν_α/ν_β and ν_1/ν_2 stand for the flavor and mass eigenstates, respectively. The plane wave description of a mass eigenstate can be expressed as $|\nu_i(t) >= e^{-i(E_i t - \vec{p_i}\vec{r})/\hbar}|\nu_i(0)>$, where E_i is the energy of the mass eigenstate i, t is the propagation time, $\vec{p_i}$ is the neutrino momentum and $\vec{r}$ is the position relative to the origin, i.e. the production point by default.
   (a) Assuming the ultra-relativistic case, $E_i \sim p_i c >> m_i c^2$, show that $|\nu_i(L) >= e^{-im_i^2 c^3 L/(2E\hbar)}|\nu_i(0)>$, where L stands for the propagation distance from origin and E is the measured neutrino energy (5%).
   (b) Find the flavor changing probability $P_{\alpha\to\beta}$ as a function of L, m_1, m_2, θ and E, assuming the tagged flavor is α at $t = 0$ (10%).

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