題號： 103
科目：微分方程
節次： 2

國立臺灣大學 105 學年度碩士班招生考試試題

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※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

1. (20 pts) Find the general solution $(x(t), y(t))$ of the system
$$\begin{cases} x'(t) + y(t) = 0, \\ y''(t) - x(t) = 0. \end{cases}$$

2. (20 pts) Let $x(t)$ satisfy $x'' - 4x^3 + 4x = 0$.
(a) Show that $\frac{1}{2}[x'(t)]^2 - x^4(t) + 2x^2(t) = $ constant.
(b) Assume $x(0) = 1, x'(0) = 0$. Find $x(t)$.

3. (20 pts) Let $x(t)$ is a C^1 function on $\mathbb{R}$ with $x(0) = 0$.
(a) Prove that $x(t) = 0$ for all t if $0 \le x'(t) \le x(t)$.
(b) Prove that the same conclusion $x(t) = 0$ for all t holds under the weaker assumption $-|x(t)| \le x'(t) \le |x(t)|$.

4. (20 pts)
(a) Suppose $x' + x = h(t)$, $x(0) = 0$. Find a function $A(t)$ such that
$$x(t) = \int_0^t A(t-s)h(s)\,ds.$$
(b) Suppose $y'' + y = h(t)$, $y(0) = 0, y'(0) = 0$. Find a function $B(t)$ such that
$$y(t) = \int_0^t B(t-s)h(s)\,ds.$$

5. (20 pts)
(a) Show that $\tanh'(t) = 1 - \tanh^2(t)$.
(b) Let $x(t) = p + q\tanh(\frac{t}{2})$. Find $p, q, c \in \mathbb{R}$ such that $x'' + cx' + 2x(x - \frac{1}{4})(1 - x) = 0$.

試題隨卷繳回