For Questions 1 to 7, show your calculations **in detail** on the answer sheet.

1. (10%) Evaluate the limit $\lim_{x\to 0}\frac{e^{3x}-1}{x}$.

2. (10%) Find the area of the region bounded by the graphs of $f(x)=-x^2+2x$ and $g(x)=3x^3-x^2-10x$.

3. (10%) Evaluate $\int\frac{sin\sqrt{x}}{\sqrt{x}}dx$.

4. (10%) Find the values of x for which the power series is convergent: $\sum_{n=1}^{+\infty}(-1)^{n+1}\frac{2^n x^n}{n3^n}$.

5. (10%) Find $\frac{dy}{dx}$ if $y=tanh^{-1}(cos2x)$.

6. (35%) Let w_T denote the terminal wealth of a uni-dollar investment after T periods. Consider two possible cumulative distribution functions (CDFs) for w_T as follows.

$$\begin{cases}\ln w_T \sim N(T\mu_F, T\sigma_F^2) & \text{if the CDF } F(w_T) \text{ is considered}\\ \ln w_T \sim N(T\mu_G, T\sigma_G^2) & \text{if the CDF } G(w_T) \text{ is considered'}\end{cases}$$

where $N(\mu,\sigma^2)$ represents the normal distribution with the mean to be μ and the variance to be σ^2. For any utility function $U'(\cdot)\geq 0$, the expected utility under F or G is

$$E_P U(w_T)=\int_0^{\infty}U(w_T)dP(w_T) \text{ for } P=F \text{ or } G.$$

Last, $\Phi(c)\equiv\int_{-\infty}^{c}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$ is the CDF for the standard normal distribution.

6.1 (5%) Find w_T^* such that $F(w_T^*)=G(w_T^*)$.

6.2 (10%) Suppose that $\sigma_F>\sigma_G$. Prove $\int_0^{w_T^*}[G(w_T)-F(w_T)]dw_T<0$.

6.3 (20%) Consider a utility function

$$U(w_T)=\begin{cases}w_T & \text{if } w_T\leq M\\ M\left[\ln\left(\frac{w_T}{M}\right)+1\right] & \text{if } w_T>M'\end{cases}$$

where M is a positive constant, and the expected utility under F is

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$$E_F U(w_T) = \int_0^M U(w_T)dF(w_T) + \int_M^\infty U(w_T)dF(w_T) = I + II.$$

6.3.1 (5%) Prove that $0 < I < M\Phi(\frac{\ln M - T\mu_F}{\sqrt{T}\sigma_F})$.

6.3.2 (10%) If II can be expressed as $M[\alpha\Phi(\beta) + \gamma e^{-\delta^2/2}]$, what are α, β, γ, and δ?

6.3.3 (5%) What is $\lim_{T\to\infty} \frac{1}{T} E_F U(w_T)$?

7. (15%) Define $\Phi(c) \equiv \int_{-\infty}^{c} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$ to be the cumulative distribution function for the standard normal distribution and $a > 0$ and $b > 0$.

7.1 (5%) What is $\lim_{y\to\infty} e^{ay}\Phi(-b\sqrt{y})$ if $a > b^2/2$?

7.2 (5%) What is $\lim_{y\to\infty} e^{ay}\Phi(-b\sqrt{y})$ if $a \le b^2/2$?

7.3 (5%) What is $\lim_{y\to\infty} e^{\frac{b^2 y}{2}}\sqrt{y}\,\Phi(-b\sqrt{y})$?

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