每題 10 分，總分為 100 分

1. Evaluate the following integrals: $\int_1^\infty \frac{\log x}{x^{n+1}} dx$.

2. Find the value of $\lim_{a\to\infty} \sum_{n=1}^{\infty} \frac{1}{n} \int_a^\infty \frac{\sin 2n\pi x}{x^s} dx$ for $a>0$ and $s>0$.

3. Find the value of $\frac{2}{\pi} \int_0^\infty \frac{\sin \upsilon \cos \upsilon x}{\upsilon} d\upsilon$.

4. Evaluate the following integrals: $\int_{R^n} e^{-\|x\|^2} dx_1 dx_2 \cdots dx_n$. Here $\|x\|^2 = x_1^2 + \cdots + x_n^2$

5. Find the value of $\int_0^\infty \frac{x^2}{(x^2+4)^2(x^2+9)} dx$.

6. Find the value of $\lim_{h\to 0} \frac{1}{h} \int_5^{5+h} x^7 \sin x^2 \, dx$.

7. Find the value of $\int_{-\infty}^{\infty} \int_{y/2}^{\infty} e^{-x^2+xy-\frac{5}{4}y^2} dxdy$.

8. Prove that $\frac{d \ln x}{dx} = \frac{1}{x}$. (You can use the following equation: $\frac{de^x}{dx} = e^x$.)

9. Let $F(x) = \int_x^{2x} \ln(x^2 + t) \, dt$. Find $\frac{dF(x)}{dx}$.

10. Prove that $\frac{dx^2}{dx} = 2x$.

試題隨卷繳回