*各題答案應作答於答案卡上，否則不予計分。
*每題有一個或一個以上正確選項，完整答對(無任何選項答錯)，該題得滿分。
*每題未作答或答錯(應選而未選或不應選而選) ，該題以 0 分計算。

1. (5%) Which of the following are the possible solutions for the differential equation below? $y''+by'+cy=0$ where b and c are constant, respectively.

(A) $y=5e^{x}+3e^{4x}$
(B) $y=3x+e^{x}$
(C) $y=5e^{x}(\cos 3x+2\sin 3x)$
(D) $y=e^{-x}+xe^{-x}$
(E) $y=1+2e^{x}$

2. (5%) Solve the differential equation $y^{(4)}-13y''+36y=0$. Which of the following are the possible solutions?

(A) $y=c_1e^{3x}+c_2xe^{-3x}+c_3e^{2x}+c_4xe^{-2x}$
(B) $y=c_1e^{x}+c_2e^{-x}+c_3e^{3x}+c_4e^{-3x}$
(C) $y=c_1e^{3x}+c_2e^{-3x}+c_3e^{2x}+c_4e^{-2x}$
(D) $y=c_1xe^{x}+c_2e^{-x}+c_3xe^{3x}+c_4e^{-3x}$
(E) $y=c_1e^{3x}+c_2e^{-3x}+c_3xe^{4x}+c_4e^{-4x}$

3. (5%) Which of the following are the possible integrating factors for the differential equation

$$\left(\frac{\sqrt{x^2+2y^2}}{\frac{x}{2}}+\frac{3}{y}\right)dy+\left(\frac{\sqrt{x^2+2y^2}}{y}+\frac{3}{x}\right)dx=0\,?$$

(A) x^2+y^2
(B) x^2
(C) y^2
(D) $\frac{x}{y}$
(E) xy

4. (5%) The solution for the differential equation $y''-6y'+9y=18x^{-3}e^{3x}$ is $y=e^{3x}(a+bx+cx^{-1})$, where a, b, and c are constant, respectively. Find the value of c.

(A) 9
(B) 10
(C) 11
(D)-10
(E) -9

5. (5%) Find the Fourier series of the function $f(x)=\begin{cases}-k, & -\pi<x<0\\ k, & 0<x<\pi\end{cases}$, and use the result to derive the value

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$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}\cdots$.

(A) Fourier series : $\sum_{n=2,4,6}^{\infty}\frac{8k}{n\pi}\sin(nx)$, summation: $\frac{\pi}{8}$

(B) Fourier series : $\sum_{n=1,3,5}^{\infty}\frac{4k}{n\pi}\sin(nx)$, summation: $\frac{\pi}{8}$

(C) Fourier series : $\sum_{n=1,3,5}^{\infty}\frac{8k}{n\pi}\sin(nx)$, summation: $\frac{\pi}{4}$

(D) Fourier series : $\sum_{n=1,3,5}^{\infty}\frac{4k}{n\pi}\sin(nx)$, summation: $\frac{\pi}{4}$

(E) Fourier series : $\sum_{n=2,4,6}^{\infty}\frac{4k}{n\pi}\sin(nx)$, summation: $\frac{\pi}{4}$

6. (5%) Find the Laplace transform Y(s) with respect to the differential equation $y'+7y+12\int_0^x ydx=u(t-3)$,

where $y(0)=1$ and $u(t)=\begin{cases}0 & t<0\\ 3 & t\geq 0\end{cases}$.

(A) $\frac{4}{s+3}+\frac{-2}{s+4}+(\frac{1}{s+3}+\frac{-1}{s+4})e^{-3s}$

(B) $\frac{2}{s+3}+\frac{-4}{s+4}+(\frac{-1}{s+3}+\frac{1}{s+4})e^{-3s}$

(C) $\frac{-3}{s+3}+\frac{4}{s+4}+(\frac{1}{s+3}+\frac{-1}{s+4})e^{-3s}$

(D) $\frac{-3}{s+4}+\frac{4}{s+3}+(\frac{-1}{s+4}+\frac{1}{s+3})e^{-3s}$

(E) $\frac{4}{s+4}+\frac{-3}{s+3}+(\frac{1}{s+4}+\frac{-1}{s+3})e^{-3s}$

7. (5%) Solve the differential equation $5x^2y''+46.25y=0$. Which of the following are the possible solutions?

(A) $x^{\frac{1}{2}}[c_1\cos(4\ln x)+c_2\sin(4\ln x)]$

(B) $x^{\frac{1}{2}}[c_1\cos(3\ln x)+c_2\sin(3\ln x)]$

(C) $x^{\frac{1}{3}}[c_1\cos(3\ln x)+c_2\sin(3\ln x)]$

(D) $x^{\frac{1}{3}}[c_1\cos(4\ln x)+c_2\sin(4\ln x)]$

(E) $x^{\frac{1}{2}}[c_1\cos(3\ln x)+c_2\sin(4\ln x)]$

8. (5%) Solve the general solution of the homogeneous system $\frac{dx}{dt}=3x+2y$; $\frac{dx}{dt}=x+2y$. Which of the following are the possible solutions?

(A) $x=c_1e^{-t}+3c_2e^{4t}$; $y=-c_1e^{-t}+c_2e^{4t}$

(B) $x=c_1e^{t}+3c_2e^{-4t}$; $y=-c_1e^{t}+c_2e^{-4t}$

(C) $x=c_1e^{-t}+2c_2e^{4t}$; $y=-c_1e^{-t}+c_2e^{4t}$

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(D) $x=c_1e^{t}+2c_2e^{4t}$; $y=-c_1e^{t}+c_2e^{4t}$

(E) none of the above

9. (5%) Evaluate inverse Laplace transform $\frac{s/2+7/3}{s^2+4s+6}$. Which of the following are the possible solutions?

(A) $\frac{1}{2}e^{-2t}\cos(\sqrt{2}\,t)+\frac{2\sqrt{2}}{3}e^{-2t}\sin(\sqrt{2}\,t)$

(B) $\frac{1}{3}e^{-3t}\cos(\sqrt{2}\,t)+\frac{2\sqrt{2}}{3}e^{-2t}\sin(\sqrt{2}\,t)$

(C) $\frac{1}{2}e^{-3t}\cos(\sqrt{2}\,t)+\frac{2\sqrt{2}}{5}e^{-3t}\sin(\sqrt{2}\,t)$

(D) $\frac{1}{3}e^{-2t}\cos(\sqrt{2}\,t)+\frac{2\sqrt{2}}{5}e^{-2t}\sin(\sqrt{2}\,t)$

(E) none of the above

10. (5%) Which the following are the correct description for the equation $4\frac{\partial^2 u}{\partial x^2}=3\frac{\partial^2 u}{\partial y^2}$?

(A) The equation is elliptic.

(B) The equation is parabolic.

(C) The equation is hyperbolic.

(D) The equation is nonhomogeneous.

(E) The equation is homogeneous.

11. (5%) Consider the following system of equations: $\begin{aligned} x_1-2x_2+4x_3&=5\\ 2x_1-3x_2+x_3&=3\\ 3x_1-4x_2-2x_3&=1 \end{aligned}$. The above system of linear equations is:

(A) Inconsistent

(B) Consistent with a unique solution

(C) Consistent with infinitely many solutions

(D) None of the above

12. (5%) For matrices A, B, C, D, which of the following statements are true?

(A) $AB\neq BA$

(B) $AB=0$ implies that $A=0$ or $B=0$ or $BA=0$

(C) $AC=AD$ implies $C=D$

(D) $AI=IA$, where I is identity matrix

13. (5%) Find all the eigenvalues of $A=\begin{bmatrix}2 & 2\\ 1 & 3\end{bmatrix}$.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

14. (5%) Suppose $u=(1, 2, 3, 4)$ and $v=(6, 5, -8, k)$. Find such k that makes u and v orthogonal.

(A) 1

(B) 2

(C) 4

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(D) 8

(E) -4

15. (5%) For matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 5 \end{bmatrix}$, which of the following statements are true?

(A) rank (A) = 3

(B) det (A) = 0

(C) A has an inverse.

(D) A is singular.

(E) The rows of A are linearly independent.

16. (5%) Which of the following lists of vectors in R^3 are linearly dependent?

(A) u=(1, 2, 5), v=(2, 5, 1), w=(5, 2, 1)

(B) u=(1, 2, 5), v=(1, 5, 6), w=(0, 0, 0)

(C) u=(2,-4, 8), v=(3 , -6, 12) , w=(1, 2, 6)

(D) u=(1, 2, 3), v=(4, 5, 6), w=(1, 4, 6), x=(3, 4, 6)

17. (5%) Which of the following statements are true?

(A) The mapping F: $R^2 \rightarrow R^2$ defined by F(x,y)=(xy, x) is linear.

(B) The mapping F: $R^2 \rightarrow R^2$ defined by F(x,y)=(x+y, x) is linear.

(C) A mapping F : V $\rightarrow$ U has an inverse if and only if F is one-to-one and onto.

(D) A mapping F: V $\rightarrow$ U is said to be onto if different elements of A have distinct images.

(E) Let F: V $\rightarrow$ U be a linear mapping. The kernel of F is a subspace of U and the image of F is a subspace of V.

18. (5%) Let $v1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, v1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$. Span {v1, v2, v3} is

(A) a point

(B) a line

(C) a plane

(D) R^3

19. (5%) Which of the following statement is NOT true?

(A) P is orthogonal if and only if P^T is orthogonal.

(B) If P is orthogonal, then P^{-1} is orthogonal.

(C) If P and Q are orthogonal, then PQ is orthogonal.

(D) If P is orthogonal, det(P)=0.

(E) P is an orthogonal matrix. $\|Pu\| = \|u\|$ for every u $\in$ V.

20. (5%) Let $A = \begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. Which of the following statements are true?

(A) det(AB)= -21

(B) det (A+B) = -4

(C) B is diagonalizable.

(D) A is positive definite.

(E) A is invertible.

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