

每小題 5 分，請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. f is a continuously differentiable real-valued function defined on the open interval $(-1, 4)$ such that $f(3) = 5$ and $f'(x) \geq -1$ for all x . What is the greatest possible value of $f(0)$?
(A) 3
(B) 4
(C) 5
(D) 8
(E) None of the above.
2. g is a continuous real-valued function such that $3x^5 + 96 = \int_c^x g(t)dt$, where c is a constant. What is the value of c ?
(A) -32
(B) -2
(C) 2
(D) 32
(E) None of the above.
3. Let $g(x) = e^{2x+1}$ for all real x . Then $\lim_{x \rightarrow 0} \frac{g(g(x)) - g(e)}{x} =$
(A) $2e$
(B) $4e^2$
(C) $2e^{2e+1}$
(D) $4e^{2e+2}$
(E) None of the above.
4. For what positive value of c does the equation $\log x = cx^4$ have exactly one real solution for x ?
(A) $(4e)^{-1}$
(B) $4e^{-1}$
(C) $2e^4$
(D) $4e^{1/4}$
(E) None of the above.
5. $\frac{d}{dx} \int_{x^3}^{x^4} e^{t^2} dt =$
(A) $e^{x^6} (e^{x^8 - x^6} - 1)$
(B) $e^{x^6} (4x^3 e^{x^8 - x^6} - 3x^2)$
(C) $e^{x^3} (e^{x^4 - x^3} - 1)$
(D) $e^{x^3} (4x^3 e^{x^4 - x^3} - 3x^2)$
(E) None of the above.
6. What is the 19th derivative of $\frac{x-1}{e^x}$?
(A) $(18 - x)e^{-x}$
(B) $(19 - x)e^{-x}$
(C) $(20 - x)e^{-x}$
(D) $(x - 19)e^{-x}$
(E) None of the above.

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7. If y is a real-valued function such that $y' + xy = x$ and $y(0) = -1$, then $\lim_{x \rightarrow -\infty} y(x) =$

- (A) 1
- (B) ∞
- (C) $-\infty$
- (D) $\pi/2$
- (E) None of the above.

8. If a real number x is chosen at random in the interval $[0, 3]$ and a real number y is chosen at random in the interval $[0, 4]$, what is the probability that $x < y$?

- (A) $3/4$
- (B) $1/2$
- (C) $5/8$
- (D) $5/6$
- (E) None of the above.

9. Suppose a and b are positive numbers. Then $\int_0^{\infty} \frac{e^{ax} - e^{bx}}{(1+e^{ax})(1+e^{bx})} dx =$

- (A) 0
- (B) 1
- (C) $\pi/2$
- (D) $(a - b) \log 2$
- (E) None of the above.

10. Which of the following statements are true?

- I. We can find a constant C such that $\log x \leq C\sqrt{x}$ for all positive x .
 - II. We can find a constant C such that $\sum_{k=1}^n k^2 \leq Cn^2$ for all positive integer n .
- (A) I only
 - (B) II only
 - (C) I and II
 - (D) None
 - (E) None of the above.

11. Let $f(x) = \arcsin x$, then $f^{-1}(x)$ is

- (A) $\sin x, 0 \leq x \leq 1$
- (B) $\frac{1}{\sin x}, -1 \leq x \leq 1$
- (C) $\sin x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
- (D) $\frac{1}{\cos x}, -1 \leq x \leq 1$
- (E) $\cos x, 0 \leq x \leq \pi$

12. Find $\int \sqrt{3x+5} dx$.

- (A) $\frac{1}{3}(3x+5)^{3/2} + C$
- (B) $\frac{2}{9}(3x+5)^{3/2} + C$
- (C) $\frac{2}{9}(3x+5)^{-3/2} + C$
- (D) $3(3x+5)^{-1/2} + C$
- (E) $3(3x+5)^{3/2} + C$

13. Find $\lim_{x \rightarrow 0} x \cot 3x$.

- (A) 3
- (B) 0
- (C) Does not exist
- (D) $\frac{1}{3}$
- (E) None of the above

14. $\frac{d}{dx} \int_{2x}^{5x} \sqrt{2 - \cos t} dt =$

- (A) $2\sqrt{2 - \cos 5x} - 5\sqrt{2 - \cos 2x}$
- (B) $5\sqrt{2 - \cos 2x} - 2\sqrt{2 - \cos 5x}$
- (C) $2\sqrt{2 - \cos 2x} - 5\sqrt{2 - \cos 2x}$
- (D) $5\sqrt{5 - \cos 2x} - 5\sqrt{2 - \cos 2x}$
- (E) None of the above

15. The graph in the xy -plane represented by $x = 3 \sin t$ and $y = 2 \cos t$ is

- (A) a line
- (B) a parabola
- (C) an ellipse
- (D) a hyperbola
- (E) a circle

16. Find the area inside one loop of the curve $r = \sin 2\theta$.

- (A) $\frac{\pi}{8}$
- (B) $\frac{\pi}{16}$
- (C) $\frac{\pi}{2}$
- (D) $\frac{\pi}{4}$
- (E) π

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17. The interval of convergence of series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n\sqrt{n}3^n}$.

- (A) $-3 \leq x \leq 3$
- (B) $-5 \leq x \leq 1$
- (C) $-5 \leq x < 1$
- (D) $-3 \leq x < 3$
- (E) None of the above

18. The Maclaurin series of expansion of $\frac{1}{1+x^2}$ is

- (A) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$
- (B) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$
- (C) $1 + x^2 + x^4 + x^6 + \dots$
- (D) $1 - x^2 + x^4 - x^6 + \dots$
- (E) None of the above

19. Find $\lim_{x \rightarrow -1^-} \frac{1}{1+x^2}$.

- (A) $-\infty$
- (B) ∞
- (C) 1
- (D) Does not exist and neither $-\infty$ nor ∞
- (E) None of the above

20. $\int \frac{dx}{\sqrt{9-x^2}} =$

- (A) $\sin^{-1} 3x + C$
- (B) $\sin^{-1} \frac{x}{3} + C$
- (C) $\frac{1}{3} \sin^{-1} 3x + C$
- (D) $\ln|x + \sqrt{9-x^2}| + C$
- (E) $\frac{1}{3} \ln|x + \sqrt{9-x^2}| + C$