

\*每題有一個或一個以上正確選項，完整答對(無任何選項答錯)，該題得滿分，若有任一選項答錯，該題得 0 分。

1. (5%) Which of the following is the possible integrating factor for  $y' - \frac{2}{x}y = \frac{2}{5}x^4$  ?
  - (A)  $e^{-2}$
  - (B)  $x^{-2}$
  - (C)  $\ln(x^{-2})$
  - (D)  $e^{2x^{-2}}$
  - (E)  $e^{-2x}$
  
2. (5%) Find the inverse Laplace transform of the function  $F(s) = \frac{s+1}{(s+3)(s-3)}$ .
  - (A)  $\cos 3x + \sin 3x$
  - (B)  $\cosh 3x + \frac{1}{3}\sinh 3x$
  - (C)  $e^{3x}\cos 2x + e^{3x}\sin 2x$
  - (D)  $e^{3x}\cosh 2x + e^{3x}\sinh 2x$
  - (E)  $e^{3x} + e^{-3x}$
  
3. (5%) Which are the possible solutions of the differential equation  $y'' - 4y' + 4y = e^t$ ?
  - (A)  $y = e^t$
  - (B)  $y = e^{2t}$
  - (C)  $y = e^t + e^{2t}$
  - (D)  $y = te^t + e^{2t}$
  - (E)  $y = e^t + te^{2t}$
  
4. (5%) The mass-spring system is described by  $x'' + bx' + 4x = 0$ ,  $x(0) = 4$ ,  $x'(0) = 0$ .  
Under which condition, the system is in the state of underdamped motion?
  - (A)  $b > 4$
  - (B)  $0 < b < 4$
  - (C)  $b = 4$
  - (D)  $b = 2$
  - (E) None of above
  
5. (5%) Solve the equation  $y^{(5)} - y^{(4)} - 2y^{(3)} + 2y'' + y' = y$ . Which are the possible solutions?
  - (A)  $y = c_1e^x + c_2xe^x + c_3x^2e^x + c_4x^3e^x + c_5xe^{-x}$
  - (B)  $y = c_1e^x + c_2xe^x + c_3e^{-x} + c_4xe^{-x} + c_5x^2e^{-x}$
  - (C)  $y = c_1e^x + c_2xe^x + c_3e^{-x} + c_4e^{-2x} + c_5xe^{-2x}$
  - (D)  $y = c_1e^x + c_2xe^x + c_3x^2e^x + c_4e^{-x} + c_5xe^{-x}$
  - (E)  $y = c_1e^x + c_2xe^x + c_3e^x + c_4e^{2x} + c_5e^{-2x}$

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6. (5%) Which boundary value problems have nontrivial solutions? (Trivial solution is defined as  $y(x) \equiv 0$ .)
- (A)  $y'' + 2y' - 3y = 0; y(0) = 0, y'(1) = 0$
  - (B)  $y'' = 0; y(-1) = 0, y(1) - 2y'(1) = 0$
  - (C)  $y'' + y = 0; y(0) = 0, y(\frac{\pi}{2}) = 0$
  - (D)  $y'' + y = 0; y(0) = 0, y(\frac{\pi}{2}) = 1$
  - (E) All of above
7. (5%) Which functions satisfy the wave equation  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k^2} \frac{\partial^2 u}{\partial t^2}$  ?
- (A)  $u(x, t) = (x^2 + k^2)t^2$
  - (B)  $u(x, t) = \sin x \cos kt$
  - (C)  $u(x, t) = \tan^{-1}(x + kt)$
  - (D)  $u(x, t) = \ln(x + kt)$
  - (E)  $u(x, t) = e^{-kt} \sin x$
8. (5%) The initial value problem is described as follows:  $y'' + 4y = 0; y(0) = 2, y'(0) = 2$ . The Laplace transform  $Y(s)$  of the solution  $y(t)$  is:
- (A)  $\frac{2}{s+4} + \frac{2}{s^2+4}$
  - (B)  $\frac{2}{s^2+4}$
  - (C)  $\frac{2}{s^2+4} + \frac{2s}{s^2+4}$
  - (D)  $\frac{2s}{s^2+4}$
  - (E)  $\frac{2}{s+4} + \frac{2s}{s^2+4}$
9. (5%) The differential equation  $x'' + 3x' - 5x = \sin t$  is equivalent to the system:
- (A)  $x' = 3x - 5y; y' = \sin t$
  - (B)  $x' = 5x - 3y; y' = \sin t$
  - (C)  $x' = y; y' = 5x - 3y + \sin t$
  - (D)  $x' = y; y' = 3x - 5y + \sin t$
  - (E) None of above
10. (5%) The differential equation is  $y' = -\frac{2y}{x}$  in the standard form, and  $2xydx + x^2dy = 0$  in a differential form. This differential equation is
- (A) linear
  - (B) homogenous
  - (C) exact
  - (D) separable
  - (E) Bernoulli

11. (10%) Which of the following are correct?

- (A) The system defined by  $F(x,y)=(x^2, x)$  is linear.
- (B) The system defined by  $F(x,y)=(dx/dt, x)$  is linear.
- (C)  $A$  and  $B$  are  $m \times n$  matrices. If  $Aw = Bw$  for all  $w$  in  $R^n$ , then  $A = B$ .
- (D) The columns of matrix  $A$  contains zero vector. If  $Ax=b$  have solution, it will have Infinite solutions.
- (E) The zero vector of  $R^n$  is within the span of any finite subset of  $R^n$ .

12. (10%) For the matrix  $A$ , which of the following are correct?

$$A = \begin{bmatrix} 3 & 1 & -2 & 1 & 5 \\ 1 & 0 & 1 & 0 & 1 \\ -5 & -2 & 5 & -5 & -3 \\ -2 & -1 & 3 & 2 & -10 \end{bmatrix}$$

- (A) The rank of  $A$  is 4.
- (B) The rank of  $A$  is 3.
- (C) The dimension of null space of  $A$  is 0.
- (D) The dimension of null space of  $A$  is 1.
- (E) The dimension of null space of  $A$  is 2.

13. (5%) Which of the following are correct?

- (A) Below vector set  $S$  is dependent.

$$S = \left( \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right)$$

- (B) Invertible matrix must be square matrix.
- (C) Square matrix must be invertible.

(D) The inverse of matrix  $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$  is  $\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$

- (E) A  $n \times n$  matrix is invertible if its columns span  $R^n$ .

14. (10%) For the matrix below, which of the following are correct?

$$\begin{bmatrix} 3 & 1 & 0 \\ 0 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

- (A) -1 is its eigenvalue.
- (B) 1 is its eigenvalue.
- (C) -3 is its eigenvalue.
- (D) Zero vector belongs to its eigenvector.
- (E)  $\begin{bmatrix} 1 & 0 & 2 \end{bmatrix}^T$  is a basis of its eigenspace.

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15. (5%) Which of the following are correct?

- (A) An eigenvalue of a matrix has infinite number of eigenvectors.
- (B) If  $\det(A - \lambda I_n) = 0$ ,  $\lambda$  is the eigenvalue of  $n \times n$  matrix  $A$ .
- (C) If  $\lambda$  is the eigenvalue of matrix  $A$ , the columns of  $(A - \lambda I_n)$  are independent.
- (D) If  $\lambda$  is the eigenvalue of matrix  $A$ , there exist a non-zero vector  $v$  such that  $Av = \lambda v$ .
- (E) The number of the eigenvalue of  $n \times n$  matrix  $A$  may larger than  $n$ .

16. (10%) Which of the following are correct?

- (A) Any orthogonal set of vectors is linearly independent.
- (B) Any orthonormal set of vectors is linearly independent.
- (C) If  $n \times n$  matrix  $A$  is orthogonal,  $A$  is invertible.
- (D) If  $n \times n$  matrix  $A$  is orthogonal,  $AA^T = I_n$ .
- (E) If  $n \times n$  matrix  $A$  is orthogonal,  $\det(A) = 1$

試題隨卷繳回