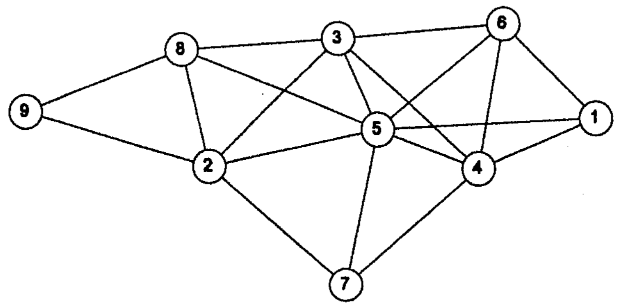
請在答案卷可作答部分的第一頁繪製十二列的表格如下並填入各題的答案

1	
2	
⋮	
11	
12	

1. (5%) Consider the following undirected graph. Write down 4 nodes which form an independent set.



2. (5%) A palindrome is a sequence of symbols that reads the same left to right as right to left. What is the number of palindromic binary numbers of length n?

3. (5%) Derive

$$A = \_\_\_\_$$
$$B = \_\_\_\_$$

in

$$\binom{2n}{n+1} + \binom{2n}{n} = \frac{\binom{A}{B}}{2}.$$

4. (10%) Derive

$$A = \_\_\_\_$$
$$B = \_\_\_\_$$

in

$$\sum_{k=1}^{n} \binom{n}{k}\binom{n}{k-1} = \binom{A}{B}.$$

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5. (10%) The solution to the recurrence equation

$$a_{n+2} = a_{n+1} + 2 \times a_n$$

is of the form:

$$a_n = A(-X)^n + BY^n.$$

Derive

$$\begin{aligned} A &= \_\_\_\_, \\ B &= \_\_\_\_, \\ X &= \_\_\_\_, \\ Y &= \_\_\_\_, \end{aligned}$$

in terms of (the arbitrary initial conditions) a_0 and a_1.

6. (10%) Consider

$$x_1 + x_2 + \cdots + x_n = r,$$

where $x_i > n_i$ for $1 \le i \le n$. The number of positive integer solutions is ____.

7. (5%) Is the following a tautology:

$$[(p \to q) \land \neg p] \to \neg q$$

Why?

8. (10%) How many of the following five statements are correct? ________

- Let V be a vector space. Let S be a subset of V. Let U be a subspace of V. If $S \subseteq U$, then $\operatorname{span}(S) \subseteq U$.
- If R is a linearly dependent subset of a vector space, then $x \in \operatorname{span}(R \setminus \{x\})$ holds for each vector $x \in R$.
- Based on any consistent axiom set for set theory, any vector space admits a basis.
- Based on the standard ZFC axioms for set theory, each inner-product space admits an orthonormal basis.
- For any complex vector spaces V and W with $\dim(W) < \infty$, if T is a linear surjection from V to W, then $\dim(V) = \operatorname{nullity}(T) + \operatorname{rank}(T)$.

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9. (10%) How many of the following five statements are correct? __________

   - The solution set for a system of homogeneous linear equations having an $m \times n$ rational coefficient matrix is a rational vector space.
   - An $m \times n$ complex matrix A is invertible if and only if A^* is invertible.
   - If A is an $m \times n$ rational matrix, then $\text{rank}(A) = \text{rank}(A^t)$.
   - If A is an $m \times n$ invertible real matrix, then $\text{nullity}(A) = \text{nullity}(A^{-1})$.
   - If A is an $n \times n$ complex matrix, then $\det(A^*) = \overline{\det(A)}$.

10. (10%) How many of the following five statements are correct? __________

    - Let A be an $n \times n$ rational matrix. If the rational n-tuple vector space $\mathbb{Q}^n$ is the direct sum of the eigenspaces of A, then A can be diagonalized.
    - If A is an $n \times n$ complex matrix with $A^*A = AA^*$, then the eigenspaces of A^* equal the eigenspaces of A.
    - If A is an $n \times n$ real matrix with $A^t = A$, then the characteristic polynomial of A can be written as a product of degree-one polynomials with real coefficients.
    - If A is an $n \times n$ complex matrix with $A^t = A$, then all eigenvalues of A are real.
    - If A and B are unitarily equivalent $n \times n$ complex matrices, then the trace of A^*A equals the trace of B^*B.

11. (10%) How many zero entries are there in the inverse of the following matrix? _____

$$\begin{pmatrix} -15 & -6 & 5 & 9 \\ -12 & 9 & 4 & -2 \\ 20 & 8 & 1 & -12 \\ 18 & -2 & -6 & 3 \end{pmatrix}$$

12. (10%) Give a basis for the vector space of the linear transformations from the vector space $\mathbb{R}^3$ of real triples to the vector space $\mathbb{R}^2$ of real pairs: ____________________.

試題隨卷繳回