1. Consider a binary transmission in which symbol 1 is represented by $s_1(t)$, $0 \le t \le T$ and symbol 0 is represented by $s_2(t)$, $0 \le t \le T$. Assume that the probabilities of transmission of 1 and 0 are both 0.5.

(a) (4%) Show the impulse response of the matched filter. No proof is needed.

(b) (4%) In practice, how can we realize the matched filter?

2. Consider a communication system in which the transmitted signal is $x_s(t) = \Sigma_k a_k\delta(t - kT_b)$, where $a_k \in \{+1,-1\}$ and T_b is the symbol interval. The output of the transmitter is $x_T(t) = x_s * h_T(t)$, where $h_T(t)$ is the transmit filter and $*$ denotes the convolution operation. After passing through a channel with impulse response $h_C(t)$ and the addition of Gaussian noise $n(t)$ and further passing through a receive filter with impulse response $h_R(t)$, we have the signal

$$v(t) = \sum_k A a_k P_r(t - t_d - kT_b) + n_0(t)$$

where $AP_r(t-t_d) = h_T * h_c(t) * h_R(t)$, $n_o(t) = n(t) * h_R(t)$, $n(t)$ is the additive white Gaussian noise.

(a) (5%) What is the intersymbol interference (ISI) for the sampled output of $v(t)$ at $t_m = mT_b + t_d$?

(b) (5%) What is the desired form of the Fourier Transform of $P_r(t)$ by which the receiver can eliminate the ISI?

(c) (5%) Describe any other method for the receiver to avoid the effect of ISI by not eliminating ISI.

3. (7 %) What is the maximum a *posteriori probability* (MAP) detector and what is the maximum likelihood detector?

4. For a 16QAM, its symbol is

$$s(t) = \sqrt{\frac{2E_0}{T}} a\cos(2\pi f_c(t)) + \sqrt{\frac{2E_0}{T}} b\sin(2\pi f_c(t)), \quad 0 \le t \le T,$$

for $a, b \in \{-3, -1, 1, 3\}$, where $f_c = 1/T$ is the carrier frequency.

(a) (6%) Derive the average symbol energy of this 16QAM.

(b) (6%) Derive the symbol error probability of this 16QAM if maximum likelihood decision is used.

5. Consider a maximum-length binary sequence with period of $N = 2^m - 1$.

(5%) What is its periodic autocorrelation function?

(3%) Is this sequence a random sequence?

6. Illustrate the following terms: (6%)

(a) LTI system

(b) FIR filter

7. Determine the following convolutions (12%)

(a) $\text{sinc}(x) * \text{sinc}(2x)$

(b) $\cos(x) * \sin(2x)$

8. (a) Determine $f[n]$ if $f[n]$ is causal and the Z transform of $f[n]$ is (14%)

$$F(z)=\frac{z^{-1}}{1-4z^{-1}+3z^{-2}}$$

(b) Determine $f[n]$ if $f[n]$ is causal and the Z transform of $f[n]$ is

$$F(z)=z^{2}\left(\exp(-z^{-2})-1\right)$$

9. Suppose that the Fourier transform of $f(t)$ is (18%)

$$F(\omega)=1-\frac{|\omega|}{2\pi} \quad \text{for } -2\pi<\omega<2\pi, \qquad F(\omega)=0 \quad \text{otherwise.}$$

(a) Determine $f(t)$.

(b) Determine $\int_{-\infty}^{\infty} f^{2}(t)\,dt$.

(c) Determine the Fourier transform of

$$f(t)\sum_{n=-\infty}^{\infty}\delta\left(t-\frac{n}{6}\right)$$

試題隨卷繳回