1. (10 pts) Determine the slope of the tangent line to the curve $x^2y^3 + y = 2$ through the point $(1, 1)$. (Present your derivation.)

2. (10 pts) When $y = f(x)$, $f(0) = 1$, and

$$\frac{dy}{dx} = (1 + y^2)\cos(x),$$

determine $f(x)$. Present the derivation.

3. (10 pts) (a) (4 points) Give a sketch of Ω. Here, Ω is defined by $y > 0$, $y^2/16 - 4 \le x \le y^2/4 - 1$, and $1 - y^2/4 \le x \le 4 - y^2/16$.
(b) (6 points) Evaluate the integral $\int_\Omega y^2 dA$ over the region Ω.

4. (15 pts) Find the area of the region enclosed by this graph, $r = \sin 2\theta$. In your answer, it should include the sketch of the region (5 points) and determine the area (10 points). Show your work.

5. (10 pts) Compute the following limit as x goes to 1.

$$\lim_{x\to 1} \sum_{k=1}^{25} \frac{x^k - 1}{\ln(x)}.$$

6. (15 pts) Consider a triangular pyramid having a triangular base with vertices A, B, and C. Denote the opposite sides by a, b, and c in which a corresponds to vertice A, and etc. Let D be the foot of the altitude to the apex (i.e., vertex, tip) of the pyramid. Its height is h which is a fixed height.

   (a) (8 pts) Draw lines DE, DF, and DG from D perpendicular to the sides a, b, and c of the triangle, with lengths x, y, and z, respectively. Also, draw lines from D to the vertices of the triangle. Show that the area of triangle ABC, T, is equal to $(ax+by+cz)/2$. (i.e., $T = (ax+by+cz)/2$) In your solution, it should includes the plot of curve C. Determine which of these triangular pyramids has the minimum surface area.

   (b) (7 pts) Compute the lateral surface area of the minimal-area pyramid. The answer should be expressed as a function of s, h, and T. (Note that the lateral surface area does not include the area of the triangle ABC.)

7. (15 pts) Let S be the part of the plane $z = 1 - x - y$ with $x \ge 0$, $y \ge 0$, and $z \ge 0$. Evaluate the surface integral of $f(x, y, z) = xy$ over S.

8. (15 pts) Let C be the curve which consists of a line segment from $(1, 1)$ to $(4, 4)$, a semicircular arc from $(4, 4)$ to $(6, 4)$, and then a line segment from $(6, 4)$ to $(8, 4)$. Let $\mathbf{F} = (\exp(x - 2y), -2\exp(x - 2y))$.

   (a) (3 pts) Plot curve C.

   (b) (12 pts) Evaluate $\int \mathbf{F} \cdot dr$.

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