※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

In this exam,

- The height of a leaf (in either a tree or heap) is defined as 0.
- The depth of the root (in either a tree or heap) is defined as 0.
- The degree of a node (in either a tree or heap) is defined as the number of its children.
- The size of a tree/heap is defined as the total number of keys (items) it stores. Unless otherwise specified, n denotes the size of the data structure.
- All heaps, unless otherwise specified, are min heaps.

一、(30%)單選題（每題 3 分，答錯不倒扣）

Please choose the best possible answer in each of the following questions. In questions that ask for time complexities, please choose the tightest bound.

1. Consider the following five recursive functions:
   (A) $T(n) = 64T\left(\frac{n}{16}\right) + n(\log n)^4 + n\sqrt{n}(\log n)^4$
   (B) $T(n) = 27T\left(\frac{n}{8}\right) + 65n\log(\log n) + 3\sqrt{n}(\log n)^3 + \log(n!)$
   (C) $T(n) = 11T\left(\frac{n}{5}\right) + 2\log^* n$
   (D) $T(n) = T(n-1) + \sqrt{n}(\log n)^2$
   (E) $T(n) = T(\sqrt[3]{n}) + 1$
   Which recursion yields the highest asymptotic complexity for $T(n)$?
2. For an AVL tree with 2021 nodes, let x be the minimum possible height, y be the maximum height, then (y-x) mod 5 is
   (A) 0　(B) 1　(C) 2　(D) 3　(E) 4
3. For a 2-3-4 tree of height 5, let x be the minimum possible size, y be the maximum possible size, then (xy) mod 5 is
   (A) 0　(B) 1　(C) 2　(D) 3　(E) 4
4. For a binary heap of height 2021, let x be the minimum possible size, y be the maximum possible size, then (3x+y) mod 5 is
   (A) 0　(B) 1　(C) 2　(D) 3　(E) 4
5. Consider a binary heap that results from successively inserting keys 6, 3, 7, 8, 5, 1, 2, 4, 9 into an initially empty heap. Then the depth of key 5 is
   (A) 0　(B) 1　(C) 2　(D) 3　(E) None of above
6. A binomial heap contains 2021 items. How many among them have depth 7?
   (A) 165　(B) 170　(C) 175　(D) 180　(E) 185

For problems 7 to 10, please consider a graph with $|V|$ vertices and $|E|$ edges, to which the Dijkstra algorithm is applied to find the shortest path. Assume $|E|$ is both $O(|V|^2)$ and $\Omega(|V|)$.

7. If Dijkstra algorithm is implemented with Binomial heap as priority queue, then the complexity is
   (A) $O(|V|^2+|E|\log|V|)$　(B) $O(|V|^2)$　(C) $O(|E|\log|V|)$　(D) $O(|E|+|V|\log|V|)$　(E) $O(|E|)$
8. If Dijkstra algorithm is implemented with Fibonacci heap as priority queue, then the complexity is
   (A) $O(|V|^2+|E|\log|V|)$　(B) $O(|V|^2)$　(C) $O(|E|\log|V|)$　(D) $O(|E|+|V|\log|V|)$　(E) $O(|E|)$
9. If Dijkstra algorithm is implemented with binary heap as priority queue, then the complexity is
   (A) $O(|V|^2+|E|\log|V|)$　(B) $O(|V|^2)$　(C) $O(|E|\log|V|)$　(D) $O(|E|+|V|\log|V|)$　(E) $O(|E|)$
10. If Dijkstra algorithm is implemented with doubly-linked list as priority queue, then the complexity is
   (A) $O(|V|^2+|E|\log|V|)$　(B) $O(|V|^2)$　(C) $O(|E|\log|V|)$　(D) $O(|E|+|V|\log|V|)$　(E) $O(|E|)$

二、(70%)複選題（每題 5 分，每答錯一個選項倒扣 1 分至該題 0 分止）

11. Which of the following statements are true?
   (A) $\lceil \log n \rceil! = O(n^2)$
   (B) $(\log n)^n = O(n!)$
   (C) $\log(n!) = O(n\log n)$
   (D) $\log^* n = O(\log^* \log n)$
   (E) $3^{\log_9 n} = O(\sqrt{n})$

12. Suppose $f(n)$ is a positive function, which of the following statements are true?
(A) If $f(n) = O(n^2)$, then $\lim_{n\to\infty} \frac{f(n)}{n^2}$ exists and that $\lim_{n\to\infty} \frac{f(n)}{n^2} < \infty$.
(B) If $f(n) = \Theta(n^3)$, and that $\lim_{n\to\infty} \frac{f(2n)}{f(n)}$ exists, then $\lim_{n\to\infty} \frac{f(2n)}{f(n)} = 8$.
(C) If $f(n) = \Theta(\sqrt{n})$, then $f(n)^2 = \Theta(n)$.
(D) If $f(n) = \Omega(n \log_2 n)$, then $2^{f(n)} = \Omega(n!)$.
(E) If $f(n) = \Omega(n!)$, then $\log f(n) = \Omega(n \log n)$.
13. Which of the following statements are true?
(A) All binary heaps are leftist heaps.
(B) All leftist heaps are of height $O(\log n)$.
(C) The Insert() operation in a leftist heap takes $O(\log n)$ time.
(D) The Delete() operation in a leftist heap takes $O(\log n)$ time.
(E) The Merge() operation between leftist heaps takes $O(\log n)$ time.
14. Which of the following statements are true?
(A) Stack implements a FIFO (first-in-first-out) policy.
(B) Queue implements a LIFO (last-in-first-out) policy.
(C) Stack can be realized by deque (double-ended queue).
(D) The Find() operation in a doubly linked list takes O(n) time.
(E) The Enqueue() operation in a queue takes O(1) time.
15. Consider a binary search tree that results from successively inserting keys 2, 8, 9, 1, 5, 7, 6, 3, 4 into an initially empty tree. Which of the following statements are true?
(A) The tree height is 4.
(B) Key 8 is prior to key 3 in terms of pre-order traversal.
(C) Key 2 is prior to key 7 in terms of post-order traversal.
(D) Key 4 is prior to key 5 in terms of in-order traversal.
(E) Key 3 and key 9 are siblings.
16. Which of the following are balanced search trees?
(A) Binary search tree
(B) Red-black tree
(C) AVL tree
(D) 2-3-4 tree
(E) Splay tree
17. Which of the following statements are true?
(A) An insertion in AVL tree requires $\Omega(\log n)$ single- and double-rotations in the worst case.
(B) An insertion in AVL tree requires $O(1)$ single- and double-rotations in the worst case.
(C) A deletion in AVL tree requires $\Omega(\log n)$ single- and double-rotations in the worst case.
(D) A deletion in AVL tree requires $O(1)$ single- and double-rotations in the worst case.
(E) There is no red left child in AA tree.
18. Consider an AVL tree that results from successively inserting keys 2, 4, 5, 3, 8, 7, 1, 6, 9 into an initially empty tree. Which of the following statements are true?
(A) The tree height is 4.
(B) Key 3 and key 5 have the same depth.
(C) Key 2 and key 8 have the same height.
(D) Key 6 is a leaf.
(E) Key 7 is the parent of key 9.
19. Consider the splay tree in Figure 1. Which of the following statements are true?
(Assume bottom-up splay, also assume the Join() operation splays on the maximum element in the left tree, which then attaches the right tree.)
(A) In Figure 1, after deleting key 26, then key 33 is a child of key 48.
(B) In Figure 1, after deleting key 26, then key 17 is the root.
(C) In Figure 1, after inserting key 75, then key 64 is an ancestor of key 99.
(D) In Figure 1, after inserting key 75, then key 33 is a descendant of key 55.
(E) In Figure 1, after inserting key 75, then key 75 is the root.

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20. Consider the red-black tree in Figure 2, which of the following statements are true? (Assume bottom-up insertion/deletion)
 (A) In Figure 2, after inserting key 14, then there are 6 red nodes.
 (B) In Figure 2, after inserting key 54, then key 49 and key 54 are siblings.
 (C) In Figure 2, after deleting key 22, then key 25 and key 73 are siblings.
 (D) In Figure 2, after deleting key 66, then key 35 is red.
 (E) In Figure 2, after deleting key 83, then key 89 is red.
21. Consider an AA tree that results from successively inserting keys 96, 49, 79, 14, 41, 87, 75, 90, 61 into an initially empty tree. Which of the following statements are true?
 (A) The root is key 61.
 (B) There is less than 3 horizontal links.
 (C) Key 41 and key 49 are in the same level.
 (D) Key 87 and key 96 are siblings.
 (E) Key 14 is the parent of key 41.
22. Consider a min-heap. Which of the following statements are true?
 (A) For any subtree of a min-heap, the root of the subtree contains the smallest key occurring anywhere in that subtree.
 (B) An array that is in ascending order is a binary min-heap.
 (C) The sequence [1,5,6,7,14,17,10,23,13,12] is a binary min-heap.
 (D) The largest key must reside in a leaf.
 (E) The smallest key must reside in a leaf
23. Consider a Fibonacci heap that results from successively inserting 929 (distinct) keys into an initially empty heap, followed by deleting the minimum key. Which of the following statements are true?
 (A) The Fibonacci heap is of degree 9.
 (B) The Fibonacci heap is of height 12.
 (C) There are 5 trees in the Fibonacci heap.
 (D) With one additional decrease key operation, the Fibonacci heap can have 6 trees.
 (E) With two additional decrease key operations, the Fibonacci heap can have 7 trees.
24. Which of the following are desirable properties of a hash function $h(x)$?
 (A) If $x_1, \ldots, x_n$ are the items to be hashed, then the numbers $h(x_1), \ldots, h(x_n)$ should be uniformly distributed over the integers.
 (B) The range of $h(x)$ should include a wide range of integers.
 (C) The range of $h(x)$ should stay within the desired hash-table size.
 (D) It should be computable in $O(1)$ time.
 (E) It should take distinct values over all possible items to be hashed.

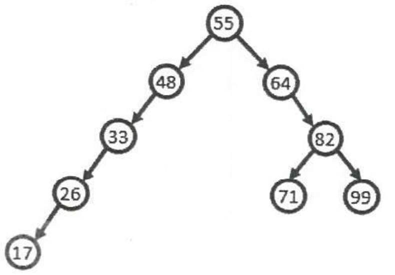

Figure 1: Splay tree

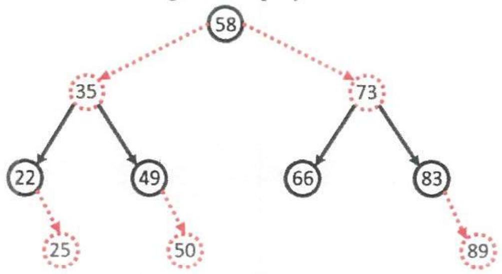

Figure 2: Red-black tree

試題隨卷繳回