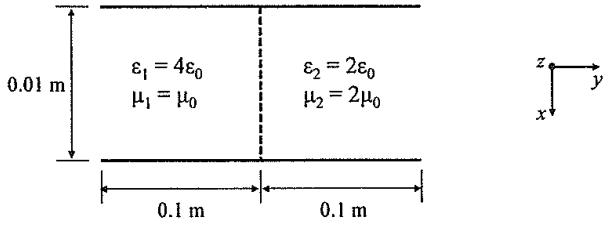
請於答案卷上非選擇題作答區標明題號作答。計算題請詳列過程。

$\varepsilon_0 = 10^{-9}/(36\pi)$ F/m, $\mu_0 = 4\pi\times10^{-7}$ H/m

1. Consider two uniform plane waves propagating in free space. The two waves are denoted by ① and ②, respectively. The electric field of wave ① is $\mathbf{E} = E_0 \cos(8\pi\times10^{12}\, t - \beta_1\times y)\, \mathbf{a}_z$. The electric field of wave ② is $\mathbf{E} = 2E_0 \cos(2\pi f_2\times t - 2\pi\times10^4\, z)\, \mathbf{a}_y$. The speed of light in free space is 3×10^8 m/s.
   (a) (5%) Which wave propagates in the z direction?
   (b) (5%) Which wave has an electric field that is along the z direction?
   (c) (5%) Find β_1 (in rad/m)?
   (d) (5%) Find f_2 (in Hz)?
   (e) (5%) What is the wavelength of wave ①?
   (f) (5%) What is the wavelength of wave ②?
   (g) (5%) If the time-averaged power flow density of wave ① is P_1, what is the time-averaged power flow density of wave ②?

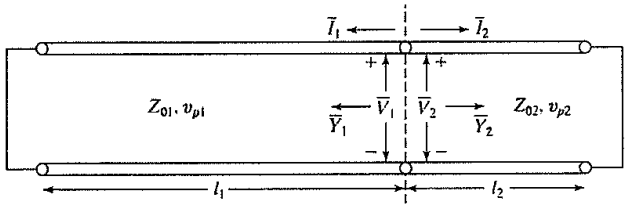
2. (15%) A parallel-plate transmission line consists of an arrangement of two perfect dielectrics, as shown by the transverse cross section in the figure below. Note that $\mu_1\varepsilon_1 = \mu_2\varepsilon_2$, so that the TEM waves propagating in the two dielectrics are in phase at all points along the interface between the dielectrics. Neglect fringing of fields and compute the values of L, C, and Z_0 of the line. (C is the capacitance per unit length; L is the inductance per unit length.)



3. (10%) The input impedance of a short-circuited transmission line of length l is

$$\bar{Z}_{in} = jZ_0 \tan\beta l = jZ_0 \tan\frac{2\pi f}{v_p} l$$

A resonant system is formed by connecting two short-circuited transmission lines as follows



At the junction, $\bar{V}_1 = \bar{V}_2$ and $\bar{I}_1 + \bar{I}_2 = 0$. Please show that

$$Z_{01} \tan\frac{2\pi f}{v_{p1}} l_1 + Z_{02} \tan\frac{2\pi f}{v_{p2}} l_2 = 0$$

見背面

4. The cutoff frequency of the mode of a rectangular waveguide (with dimensions a and b) is

$$f_c = \frac{v_p}{\lambda_c} = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2}$$

m or $n = 0, 1, 2,\ldots$, for TE_{mn} modes (but not both 0) and m or $n = 1, 2,\ldots$, for TM_{mn} modes.

(I) Please determine the lowest five modes for the following cases:

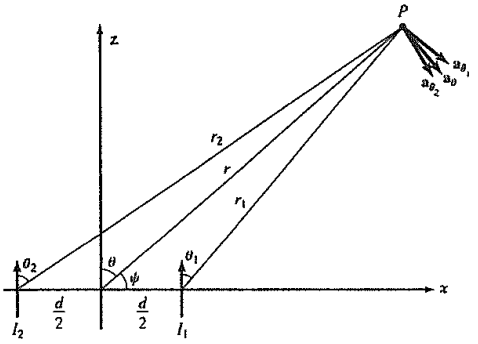
(i) b/a=1 (5%); (ii) b/a=1/2 (5%); (iii) b/a=1/3 (5%).

(II) (5%) Given a = 3 cm and free space for the dielectric in the waveguide, please find the propagating modes for f = 9000 MHz in the case (i).

5. For the radiation field of two Hertzian dipoles, the electric fields of the individual dipoles are

$$\mathbf{E}_1 = -\frac{\eta\beta I_0\, dl \sin\theta_1}{4\pi r_1}\sin\left(\omega t - \beta r_1 + \frac{\alpha}{2}\right)\mathbf{a}_{\theta_1}$$

$$\mathbf{E}_2 = -\frac{\eta\beta I_0\, dl \sin\theta_2}{4\pi r_2}\sin\left(\omega t - \beta r_2 - \frac{\alpha}{2}\right)\mathbf{a}_{\theta_2}$$



For $r >> d$, so that $\theta_1 \approx \theta_2 \approx \theta$ and $\mathbf{a}_{\theta_1} \approx \mathbf{a}_{\theta_2} \approx \mathbf{a}_\theta$

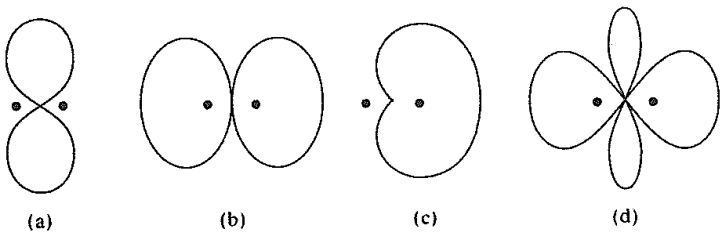
$$r_1 \approx r - \frac{d}{2}\cos\psi$$

$$r_2 \approx r + \frac{d}{2}\cos\psi$$

(i) (10%) Please show that the total field $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$ is equal to

$$-\frac{2\eta\beta I_0\, dl \sin\theta}{4\pi r}\cos\left(\frac{\beta d\cos\psi + \alpha}{2}\right)\sin(\omega t - \beta r)\,\mathbf{a}_\theta$$

(ii) (10%) Please write down the values of d and α for each case below.



試題隨卷繳回