※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

(1) (12 %) Illustrate the following four terms:
(a) aliasing effect; (b) LTI; (c) wide-sense stationary random process; (d) additive white Gaussian noise

(2) (6 %) Suppose that the continuous-time Fourier transform of $x(t)$ is $X(j\omega)$.
What is the Fourier transform of $\mathcal{Re}\{x(-2t+3)\}$ where $\mathcal{Re}\{\ \}$ means taking the real part?

(3) Determine the following convolutions
(a) (6 %) $2^{-n}u[n] * 3^{-n}u[n]$ (* means the discrete convolution and $u[n]$ is the unit step function)
(b) (6 %) $[\cos(\pi t) + \sin(4\pi t) + \cos(10\pi t)] * \text{sinc}(2t)$ (* means the continuous convolution)

(4) Suppose that $x(t)$ is real and has the bandwidth of W and
$y(t) = 3x^2(t) * x(-2t) * x(4t) + x(t)$ where * means convolution.
(a) (5 %) What is the bandwidth of $y(t)$?
(b) (3 %) How do we sample $y(t)$ properly to avoid the aliasing effect?

(5) Find the inverse Z transform of the following functions (suppose that $x[n]$ is causal).
(a) (6 %) $\dfrac{6}{6-5z^{-1}+z^{-2}}$
(b) (6 %) $\exp\left(-2z^{-2}\right)$

(6) (25%) Consider a discrete memoryless source (DMS) with the associated source alphabet $\mathcal{X} := \{a, b, c, d, e, f, g\}$. Each source output is independently selected from $\mathcal{X}$ with the probability distribution P_X given in Table 1. The goal of source coding for the DMS is to construct a code C that assigns each symbol $x \in \mathcal{X}$ a bit string $C(x) \in \{0, 1\}^{l(x)}$ with length $l(x) \in \mathbb{N}$. Its average length is defined as $\bar{L} := \Sigma_{x\in\mathcal{X}} P_X(x)l(x)$. Moreover, the code is usually required to be *uniquely decodable*.

Table 1. A table of symbols in the source alphabet $\mathcal{X}$ and their probabilities of occurrence.

Symbol	Probability P_X
a	0.2
b	0.1
c	0.005
d	0.2
e	0.3
f	0.045
g	0.15

Please answer the following questions.
(a) (5%) What does the unique decodability mean? Assume that a code has the property that $C(x) \neq C(\tilde{x})$ for each $x \neq \tilde{x}$. Is the code uniquely decodable?
(b) (5%) A class of codes called the *prefix-free codes* can be decoded with no delay (hence sometimes also called *instantaneous codes*). What is the definition for the prefix-free codes?
Every prefix-free code for the alphabet $\mathcal{X}$ with its codeword lengths $\{l(x) : x \in \mathcal{X}\}$ must satisfy the *Kraft*

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inequality. What is the Kraft inequality?

Assume that a code with its codeword lengths satisfies the Kraft inequality. Is it uniquely decodable?

Does a uniquely decodable code have to satisfy the Kraft inequality?

(c) (10%) Use the Huffman algorithm to construct a prefix-free code for the above mentioned DMS. What is the associated average length $\bar{L}$? Clearly state your coding strategy.

(d) (5%) Explain why the Huffman algorithm gives an optimal source code, i.e. no other source coding strategy has smaller $\bar{L}$.

(7) (25%) Consider a standard M-ary digital modulation scheme where the signal space diagram is spanned by two orthonormal waveforms, i.e. each symbol on the constellation has the in-phase and quadrature-phase components. Assume that the underlying channel model is additive white Gaussian noise (AWGN) with the power spectral density $N_0/2$, and that no inter-symbol interference is happening. Further, assume that all symbols on the constellation are equally probable.

Please answer the following questions.

(a) (5%) Using the signal space diagrams given in Figures 1a and 1b, respectively. Which one has the better performance (i.e. the smallest symbol error probability)? Explain your reason.

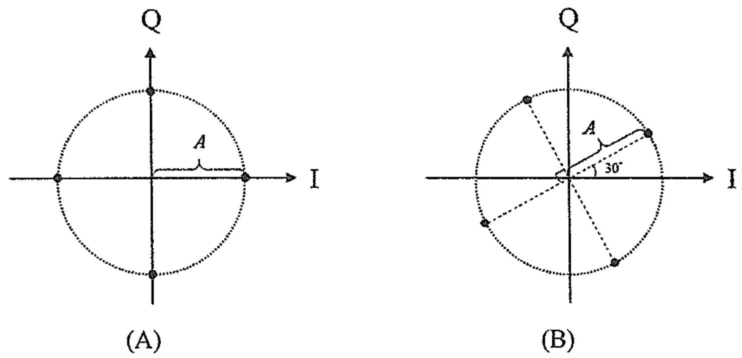

Figure 1. Signal space diagrams for a quaternary phase-shift keying (QPSK) and a $\frac{\pi}{6}$-QPSK.

(b) (10%) Derive the optimal *symbol error probability* when using the signal space diagram given in Figure 1b in terms of the average symbol energy, N_0, and the Q-function,

$$Q(u) := \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\frac{t^2}{2}} dt$$

(c) (5%) Given an arbitrary M-ary modulation scheme with the minimum distance d_{min}, derive a union upper bound for the symbol error probability in terms of d_{min} and the Q-function.

(d) (5%) Since the minimum distance of a constellation dominates the performance of a modulation scheme, one would desire a signal diagram with the same minimum distance d_{min} constraint that has *smaller* average symbol energy. Please design a 7-ary signal diagram on the I/Q-constellation with $d_{min} = 2$ and its average symbol energy no larger than 3.43.