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科目：常微分方程
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國立臺灣大學 110 學年度碩士班招生考試試題

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Ordinary Differential Equations

1. Set
$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \tag{0.1}$$

and $X(t) = (x_1(t), x_2(t))^T$.

(a) (10 points) Calculate $e^{tA} := I + \sum_{j=1}^{\infty} \frac{1}{j!} t^j A^j$, where I is the two by two identity matrix.

(b) (10 points) Solve the differential system

$$\begin{cases} x_1'(t) = x_1(t) + x_2(t) + e^t, \\ x_2'(t) = x_2(t), \end{cases} \tag{0.2}$$

with initial condition $X(0) = (2, 0)^T$.

2. (20 points) Solve the differential equation

$$x'''(t) + x''(t) - 9x'(t) - 9x(t) = 0,$$

with initial condition $x(0) = x'(0) = 0$, $x''(0) = 1$.

3. (20 points) Suppose that $f(t)$ and $g(t)$ are two solutions of the differential equation

$$x'(t) + (5 + \cos t)x(t) = \sin t. \tag{0.3}$$

Show that if $f(0) > g(0)$, then $f(t) > g(t)$ for all $t > 0$.

4. Consider the differential equation

$$x'(t) + (4 + \sin t)x(t) = \sin t. \tag{0.4}$$

Let $\phi(t)$ be the periodic solution of (0.4).

(a) (10 points) Find the value of $\phi(3\pi)$.

(b) (10 points) Let $X(t)$ be any other solution of (0.4). Show that

$$\lim_{t \to \infty} |X(t) - \phi(t)| = 0.$$

5. (20 points) Consider the differential equation

$$x''(t) + \sin x = 0. \tag{0.5}$$

Let $f(t)$ be the solution of (0.5) with initial condition $f(0) = \frac{\pi}{2}$ and $f'(0) = 1$. Find the maximum value and the minimum value of $f(t)$.

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