※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. Mass conservation (20 points)

(a) Write down the statement of mass conservation (i.e. the continuity equation) for fluid flow and explain the physical meaning of each terms. (6 points)

(b) If the fluid of interest is nearly incompressible, the mass conservation in (a) can be simplified. Write down this simplified equation, provide justifications for your simplification, and explain the physical meaning. (7 points)

(c) Consider a steady, incompressible, three-dimensional flow. The x and z velocity components are given by $u = ax^2 + by^2 + cz^2$ and $w = axz + byz^2$, where a, b, c are constants. Find the y velocity component (v) as a function of x, y, and z. (7 points)

2. Momentum conservation (we consider Newtonian fluids for all of the questions below) (25 points)

(a) What is the Newtonian fluid? (5 points)

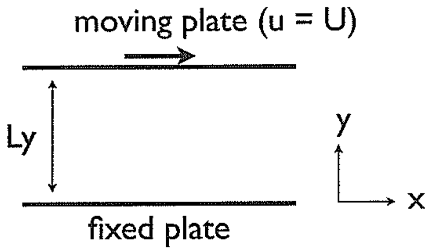
(b) Write down the differential form of momentum conservation (i.e. the Navier-Stokes equation) for incompressible flows. Interpret the physical meaning of each terms. (9 points)

(c) What is the Reynolds number? Interpret its physical meaning and its significance in characterizing flow properties. (6 points)

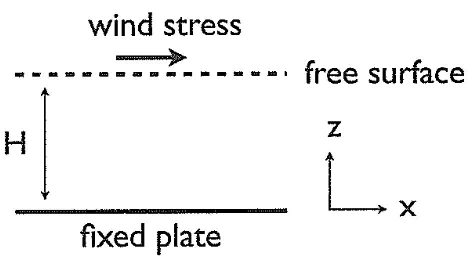
(d) Consider a fully developed pipe flow, describe the differences in time-mean velocity structure (e.g. velocity across the pipe) between laminar and turbulent flow regimes. (5 points)

3. Applications (30 points)

Fig. 1 (a)



(b)



(a) As illustrated in Fig. 1a, consider a steady, incompressible, laminar flow (with a constant viscosity) confined by two infinite parallel plates. One plate is moving at speed U, while the other plate is held fixed. Assuming that the flow is parallel ($v = 0$) and is purely two-dimensional (in x and y direction). The boundary conditions are (1) at $y = 0$, $u = v = w = 0$. (2) at $y = Ly$, $u = U$, $v = w = 0$.

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Based on the above assumptions, develop the simplified continuity and momentum equations (please provide reasons for why certain terms can be neglected) (7 points)

(b) Following (a), solve for the velocity field and estimate the shear force per unit area acting on the fixed plate. (8 points)

(c) Let's flip the dimension to x-z plane and remove the moving plate so that the upper boundary is now free surface (Fig. 1b). Wind forcing is applied at the free surface. Assuming that the wind-driven flow is <u>steady, laminar, two dimensional, and fully developed in x direction (i.e. no gradients in x)</u>. The flow is subject to the following boundary conditions: (1) at $z = 0$, $u = v = w = 0$. (2) at $z = H$, the viscous stress (τ) is only non-zero in x direction, with $\tau_{zx}(z = H) = \tau_W$.

Express the momentum balance and solve for the velocity profile ($u(z)$) (9 points)

(d) Following (c), express and sketch the profile of stress $\tau(z)$. Interpret the force balance. (6 points)

4. Vorticity (25 points)

(a) What is the definition of vorticity? What fluid properties does vorticity represent/ measure? (7 points)

(b) What is irrotational flow? (5 points)

(c) Consider the following steady, two-dimensional (in x-y plane), circular flows. The velocity components are expressed in cylindrical coordinate (in 2D $r-\theta$ plane; r is radial distance, θ is the angle from the x-axis)

(1) Flow in solid-body rotation: $u_r = 0$ $u_\theta = \omega r$ (ω is a constant)

(2) Line vortex: $u_r = 0$ $u_\theta = k/r$ (k is a constant)

Sketch the velocity field and calculate the vorticity. (8 points)

(d) Are the circular flows described above irrotational? Follow a fluid parcel in these flow fields, comment on how the parcel rotates. (5 points)

[Transformations of velocity components between the Cartesian and polar coordinates]

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_r \\ u_\theta \end{pmatrix}$$

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