※禁止使用計算機

(1) (15 pts) Let $f(x), g(x), h(x)$ be differentiable functions on $\mathbb{R}$. Consider the function

$$F(x): = \int_0^{h(x)} f(t)g(t)dt.$$

Determine $\frac{d}{dx}F(x)$.

(2) (15 pts) Determine the convergence (or divergence) of the series

$$\sum_{n=2}^{\infty} \frac{\sin n}{n(\log n)^2}.$$

(3) (20 pts) Let $f(x)$ be a differentiable function on $\mathbb{R}$ such that $f(0) = 0$ and $|f'(x)| \le |f(x)|$ for all $x \in \mathbb{R}$.
 (a) Prove that $f(x) = 0$ for all $x \in [0, \frac{1}{2}]$.
 (b) Prove that $f(x) = 0$ for all $x \in \mathbb{R}$.
 (You can use part (a) as a fact to answer part (b)).

(4) (20 pts) Find the minimum and maximum of the function $f(x) = xyz$ on the surface $S = \{(x, y, z)|2x^2 + y^2 + z^2 = 4, y + z \ge 0\}$.

(5) (15 pts) Find the volume of the solid given by $x^2 + y^2 + (z-1)^2 \le 1$ and $z \ge \sqrt{x^2 + y^2}$.

(6) (15 pts) Let $\overrightarrow{F} = (x + y^3, y + x^3, z)$ be a vector field and S be the surface given by $z = 4 - x^2 - y^2, z \ge 0$ with upward orientation. Find the flux $\iint_S \overrightarrow{F} \cdot \overrightarrow{n}\, dS$.

試題隨卷繳回