國立臺灣大學106學年度轉學生招生考試試題

題號： 17
科目：微積分(A)

※禁止使用計算機

※ 注意：請於答案卷內之「非選擇題作答區」標明題號依序作答。

(1) (20%) Determine the convergence or divergence of the following series:

$$\sum_{n=1}^{\infty}\left(\frac{\cos(\pi n)}{\tanh(\pi n)}\frac{1-\cosh(\frac{1}{n})}{\sin(\frac{1}{n})}\right).$$

(2) (20%) For a second continuously differentiable function, $f(x,y,z)$, defined on the space $\mathbb{R}^3$, its Laplacian is defined to be

$$\Delta f=\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}.$$

Derive the formula of Δf in the spherical coordinate system.

(3) (20%) Suppose that $f(x)>0$ is a continuously differentiable function defined on $x\geq 1$. Let S be the surface of revolution of the graph $y=f(x)$ about the x-axis. Let $E\subset\mathbb{R}^3$ be the solid enclosed by S and $x=1$. More precisely,

$$E=\{(x,y,z)\in\mathbb{R}^3 \mid x\geq 1 \text{ and } \sqrt{y^2+z^2}\leq f(x) \text{ for each } x\geq 1\}.$$

If the surface area of S is finite, prove that the volume of E must also be finite.

(4) Let C be the curve defined by $\{(x,y)\in\mathbb{R}^2 \mid x^4+2x^2y^2+y^4-2x(x^2-y^2)=0\}$.

(a) (10%) Sketch the curve C.

(b) (10%) Calculate the area of the region enclosed by C.

(5) (20%) Let a,b,c be three positive numbers with $a>b>c$. Let Σ be the surface defined by

$$\frac{(x-b)^2}{a^2}+\frac{(y-c)^2}{b^2}+\frac{z^2}{c^2}=4.$$

Note that Σ encloses a bounded solid. Set $\vec{n}$ to be the unit outer normal with respect to that solid. Consider the vector field

$$\vec{F}=\left(\frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}}+\cos(yz),\frac{y}{(x^2+y^2+z^2)^{\frac{3}{2}}}+y,\frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}}+e^{z^2}\right).$$

Calculate the flux integral $\int_{\Sigma}\vec{F}\cdot\vec{n}\,\mathrm{d}S$.

試題隨卷繳回