※ 注意：請於試卷上「非選擇題作答區」標明題號並依序作答。

※禁止使用計算機

No Calculator is Allowed!

(1) (a) (10 pts) Find an explicit value of $\epsilon > 0$ such that for every $x \in [0,1]$

$$|\sqrt{x} - \sqrt{x+\epsilon}| < \frac{1}{200}.$$

(b) (10 pts) Find an explicit integer N such that there exists a polynomial P of degree at most N such that for every $x \in [0,1]$

$$|\sqrt{x} - P(x)| < \frac{1}{100}.$$

Hint: You can use the expansion of $\sqrt{x+\epsilon}$ in power series in $(x-1)$.

(2) (10 pts) Let $f : [0,1] \mapsto \mathbf{R}$ be monotone increasing, i.e. if $c < d$ implies $f(c) \le f(d)$. Use the definition of the Riemann integral (comparing upper and lower sums relative to a partition of [0, 1]) to prove that $\int_0^1 f(x)dx$ exists.

(3) (a) (10 pts) Evaluate $\int_0^2 \int_{y^2}^4 y\cos(x^2)dxdy$

(b) (10 pts) Let F be a vector field $F = \langle x^3, y^3, z^3 \rangle$ and Ω be the solid region in $\mathbf{R}^3$ bounded by $x^2+y^2 \ge z^2, x^2+y^2+z^2 \le 9, y \ge |x|$.
Evaluate $\int\int\int_\Omega (x^2+y^2+z^2)dV$ and find the the flux of F through the boundary surface of Ω, oriented outwards (i.e. $\int\int_{\partial\Omega} F \cdot ndS$).

(4) (10 pts) Among all planes that are tangent to the surface $xy^2z^2 = 1$, find the ones that are farthest from the origin.

(5) Let $\{a_n\}_{n=1}^\infty$ be a sequence and $b_n = \frac{1}{n^3}(\sum_{k=1}^n k^2 a_k)$.
(a) (10 pts) Prove or disprove: If $\{a_n\}_{n=1}^\infty$ converges then $\{b_n\}_{n=1}^\infty$ converges.
(b) (10 pts) Prove or disprove: If $\{b_n\}_{n=1}^\infty$ converges then $\{a_n\}_{n=1}^\infty$ converges.

(6) (a) (10 pts) Find domain of convergence of

$$\sum_{n=1}^\infty \frac{(n+x)^n}{n^{n+x}}.$$

(b) (10 pts) Does the series

$$\sum_{n=1}^\infty e^{-\frac{x}{n}} \frac{(-1)^n}{n}$$

converge uniformly on $[0,\infty)$. Prove or disprove.

試題隨卷繳回