※ 注意：請於試卷上「非選擇題作答區」標明大題及小題題號，並依序作答。

Any device with computer algebra system is prohibited during the exam.

PART 1 : Fill in the blanks.

- Only answers will be graded.
- Each answer must be clearly labeled on the answer sheet.
- 4 points are assigned to each blank.

1. (a) $\lim_{x\to\infty}(1+\frac{3}{x})^{[x]} =$ ___(1)___ , where $[x]$ is the greatest integer which is less than or equal to x.

   (b) $\lim_{x\to 0}(\cos ax)^{\frac{1}{1-\cos bx}} =$ ___(2)___ , where a, b are constants and $ab \neq 0$.

   (c) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{2n\sqrt{1-(\frac{i}{2n})^2}} =$ ___(3)___.

2. (a) $f(x) = [\ln(1+x^2)]^x$, $\frac{d}{dx}f(x) =$ ___(4)___.

   (b) $x^3 - y^2 + y^3 = x$. At $(x, y) = (0, 1)$. $\frac{d^2y}{dx^2} =$ ___(5)___.

   (c) $f(x, y, z) = \int_z^{xy} e^{\sqrt{t}}dt$. $\nabla f =$ ___(6)___.

   (d) $f(x, y) = \frac{\sin(x^2y)}{x^2+y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. The directional derivative of f along $\mathbf{u} = (\cos\theta, \sin\theta)$ at $(0, 0)$ is ___(7)___.

3. $f(x) = \begin{cases} |x| - 1, & \text{for } -1 \leq x \leq 3 \\ (x-6)^2 - 4, & \text{for } 3 < x \leq 10. \end{cases}$. $g(x) = \int_0^x f(t)dt$, for $-1 \leq x \leq 10$.

   (a) Choose correct statement(s) about $g(x)$: ___(8)___

      i. $g(x)$ is discountinuous at $x = 3$.

      ii. $g(x)$ is not differentiable at $x = 3$.

      iii. $g(x)$ is differentiable at $x = 0$ and $g'(0) = -1$.

      iv. $g(x) < 0$ for $-1 \leq x < 0$.

   (b) The local minimum values of $g(x)$ occur at $x =$ ___(9)___.

   (c) The x-coordinates of points of inflection for $y = g(x)$ are ___(10)___.

4. A vertical fence 2 m high is located 1 m away from a wall. The length of the shortest ladder that can extend from the wall over the fence to a point on the ground is ___(11)___.

5. Compute the following integrals:

   (a) $\int_0^\infty \frac{2x-3}{(x-1)(x^2+4)}dx =$ ___(12)___.

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(b) $\int \frac{dx}{\sqrt{x^2-6x}} =$ __(13)__.

(c) $\int \sin^{-1}(\sqrt{x})\, dx =$ __(14)__.

6. (a) $\iint_D y\, dA =$ __(15)__, where D is the region in the first quadrant that lies between the circles $x^2+y^2=4$ and $(x-1)^2+y^2=1$.

   (b) The volume of the solid R lying below the plane $z=3-2y$ and above the paraboloid $z=x^2+y^2$ is __(16)__.

7. (a) C is a smooth curve in the upper half plane going from $(1,1)$ to $(0,\sqrt{2})$. $\mathbf{F}(x,y)=\frac{x-3y}{x^2+y^2}\,\mathbf{i}+\frac{3x+y}{x^2+y^2}\,\mathbf{j}$. Then, $\int_C \mathbf{F}\cdot d\mathbf{r} =$ __(17)__.

   (b) S is the part of the sphere $x^2+y^2+z^2=9$ that lies above the cone $z=\sqrt{x^2+y^2}$ with upward orientation. $\mathbf{F}(x,y,z)=\frac{x\mathbf{i}+y\mathbf{j}+z\mathbf{k}}{(x^2+y^2+z^2)^2}$. Then, $\iint_S \mathbf{F}\cdot d\mathbf{S} =$ __(18)__.

8. (a) $\sum_{n=0}^{\infty} \frac{3^n}{(2n)!} =$ __(19)__.

   (b) The Maclaurin series (the Taylor series at $a=0$) for $\cos^{-1} x$ is __(20)__.

PART 2 :

- Solve the following problems. You need to write down complete arguments.
- 10 points are assigned to each problem.

1. Find the equation of the curve on the xy−plane that passes through $(3,1)$ such that for any point P on the curve, the midpoint of the tangent line at P that lies in the first quadrant is P itself.

2. $f(x,y)$ is a differentiable function. On the curve $\tan^{-1}(xy)+y=2+\frac{\pi}{4}$, f obtains local maximum at $(x,y)=(\frac{1}{2},2)$. Suppose that $f_x(\frac{1}{2},2)=2$.

   (a) Find $\nabla f(\frac{1}{2},2)$.

   (b) Assume that on another curve $\tan^{-1}(xy)+y=1.9+\frac{\pi}{4}$, f obtains local maximum at (x_0,y_0) which is near $(\frac{1}{2},2)$. Use linear approximation to estimate $f(x_0,y_0)-f(\frac{1}{2},2)$.

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