國立臺灣大學 113 學年度碩士班招生考試試題

※請將答案填入試卷內所附的表格中。

1. (10 points) Let there be z functions from $\{\text{"A"}, \text{"B"}\}^3$ to $\{1,2,3\}$. Calculate z mod 7: _____.

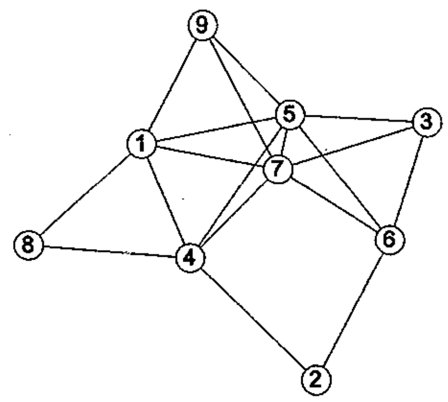
2. (10 points) There are _____ satisfying assignments to

$$(\neg w \wedge x \wedge \neg y \wedge \neg z) \vee (\neg w \wedge \neg x \wedge y) \vee (\neg w \wedge \neg x \wedge z).$$

3. (10 points) Calculate Euler's phi function (also called totient function) $\Phi(90)$: _____.

4. (10 points) Derive the solution for a_n that satisfies the recurrence equation $a_n = -a_{n-1} + 12a_{n-2}$ with $a_0 = 3$ and $a_1 = 2$: _____.

5. (10 points) For the following graph: (4 points) Its chromatic number is _____ (the smallest number of colors needed to color the nodes). (4 points) The size of the maximum independent set is _____. (1 point) Is it Eulerian _____ (yes/no)? (1 point) Is it Hamiltonian _____ (yes/no)?



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6. (10 points). Consider a system of linear equations $Ax=b$ with the following augmented coefficient matrix:

$$[A\,|\,b]=\left(\begin{array}{ccc|c}1 & 0 & -1 & 1\\ 0 & 1 & 2 & 0\\ 1 & -1 & -3 & 2\end{array}\right).$$

Let

$$S=\underset{x\in\mathbb{C}^3}{\operatorname{argmin}}\|Ax-b\|.$$

That is, S consists of the vectors $x\in\mathbb{C}^3$ that minimize the standard norm (also known as 2-norm or ℓ^2-norm) of the vector $Ax-b\in\mathbb{C}^3$. The square of the minimum standard norm of the vectors in S is ________.

7. (10 points) Consider the following sets R and S of quadruples of rational numbers:

$$R=\left\{\left(\frac{1}{3},\frac{-2}{3},\frac{1}{3},0\right),\left(\frac{-3}{2},0,\frac{3}{2},0\right)\right\}$$
$$S=\{(1,0,-1,0),(1,-1,0,0),(-1,1,0,1),(1,0,-1,1)\}.$$

If V (respectively, W) is the subspace of the rational quadruples spanned by R (respectively, S), then the subspace U of the rational quadruples satisfying the following two conditions

$W=U\oplus V$ (i.e., W is the direct sum of U and V)
$U\perp V$ (i.e., U is orthogonal to V with respect to the standard inner-product of $\mathbb{Q}$)

is ________.

8. (10 points) Let F be the field $(\{0,1,2\},+,\cdot)$, where $+$ is the mod-3 addition and $\cdot$ is the mod-3 multiplication. If the polynomial

$$p(x)=a+bx+cx^2\in\mathbb{P}(F)$$

satisfies $p(i)=i^3+2$ for each $i\in F$, then (a,b,c) is ________.

9. (10 points) If the characteristic polynomial of a linear operator T on a complex inner-product space V is

$$f_T(x)=x^4-2x^2+1,$$

then the rank(S) of the linear operator

$$S=T^6-2T^5-T^4+4T^3-T^2-2T$$

in terms of rank(T) is ________.

10. (10 points) Let U and V be vector spaces over the scalar field $\mathbb{R}$ of real numbers, where

- U consists of the 2×2 self-adjoint complex matrices and
- V consists of the 3×3 real matrices.

If

$$S=\{2,0,2,4,0,2,0,3\}$$

is a multi-set consisting of the nonnegative real numbers

$$|\sigma-1|$$

for all nonzero singular values σ of T, then the product of all singular values of the adjoint of the pseudo-inverse of T, is ________.

試題隨卷繳回