第一大題「單選題」，請依題號作答於「答案卡」(請勿作答於試卷之選擇題作答區)，未作答於答案卡者，該大題不予計分；第二大題為「非選擇題」，請作答於「試卷」之非選擇題作答區。

一、單選題

1. (8%) We are given a discrete-time system whose input $x[n]$ and output $y[n]$ are related through the following difference equation:

$$y[n] = \frac{1}{2}(x[n] + x[n-1]).$$

Determine the phase response $\Theta(\omega)$ of this system.
(A) $-\frac{\omega}{2}$、(B) $-\frac{\omega}{4}$、(C) 0、(D) $\frac{\omega}{4}$、(E) $\frac{\omega}{2}$.

2. (8%) Consider the signal:

$$x[n] = \begin{cases} a^n, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases},$$

where a is real and $|a| < \infty$, and N is a finite positive integer. Which of the following statement regarding the region of convergence (ROC) of its z-transform is true?
(A) The ROC is the entire z-plane except for $z = 0$;
(B) The ROC is the entire z-plane except for $z = \infty$;
(C) The ROC extends inwards from the innermost non-zero pole in $X(z)$ to $z = 0$;
(D) The ROC extends outwards from the outermost finite pole in $X(z)$ to $z = \infty$;
(E) Unable to decide; need information about the actual value of either a or N.

3. (8%) We can reduce the sampling rate of a sequence by a factor of M by downsampling:

$$x_d[n] = x[nM].$$

What is the frequency-domain relationship between the original signal $x[n]$ and the downsampled one $x_d[n]$?
(A) $X_d(e^{j\omega}) = X(e^{j\omega M})$;
(B) $X_d(e^{j\omega}) = X(e^{j\omega/M})$;
(C) $X_d(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\omega M - 2\pi k M)})$;
(D) $X_d(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\omega/M - 2\pi k/M)})$;
(E) Unable to decide; need information about the actual value of M to see if aliasing would occur.

4. (8%) A discrete-time *all pass filter* has a magnitude response that is unity for all frequencies, i.e., $|H(e^{j\omega})| = 1$, for all ω. We are given below the system function of five systems. Which one would correspond to an all pass filter that is *stable*?
(A) $H(z) = \frac{(z-2)(z-3)}{(z+2)(z+3)}$,
(B) $H(z) = \frac{(z-2)(z-3)}{(z+1/2)(z+1/3)}$,
(C) $H(z) = \frac{(z-2)(z+1/3)}{(z-1/2)(z+3)}$,

(D) $H(z) = \frac{(z+\frac{1}{2}-\frac{1}{2}j)(z+\frac{1}{2}+\frac{1}{2}j)}{(z-\frac{1}{2}-\frac{1}{2}j)(z-\frac{1}{2}+\frac{1}{2}j)}$;

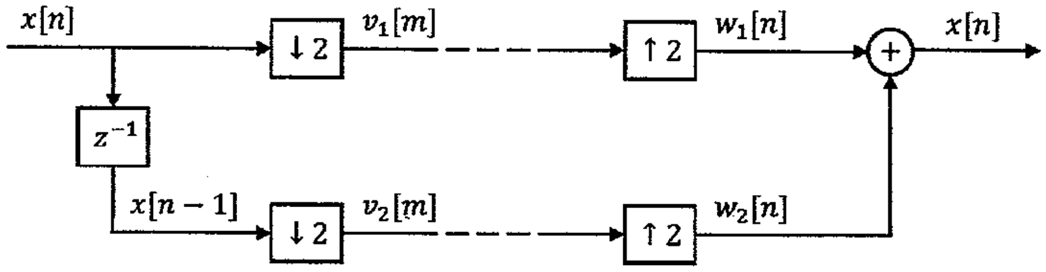
(E) None of the above.

5. (3%) We are given a filter $y(t) = \begin{cases} x(t), & x(t) \geq 0 \\ 0, & x(t) < 0 \end{cases}$. Which of the following statement is true?
(A) It's linear (L) and time-invariant (TI);
(B) It's nonlinear (NL) and time-invariant (TI);
(C) It's linear (L) and time-varying (TV);
(D) It's nonlinear (NL) and time-varying (TV).

6. (3%) We are given a filter $y(t) = x(t) * s(t) * s(t - T/2)$, where $s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$ is an impulse train with period T, and the operator '*' denotes convolution. Which of the following statement is true?
(A) It's linear (L) and time-invariant (TI);
(B) It's nonlinear (NL) and time-invariant (TI);
(C) It's linear (L) and time-varying (TV);
(D) It's nonlinear (NL) and time-varying (TV).

7. (4%) We are given a filter $y(t) = x(t)s(t)$, which represents the element-wise multiplication of the input $x(t)$ and the impulse train $s(t)$ defined above. Which of the following statement is true?
(A) It's linear (L) and time-invariant (TI);
(B) It's nonlinear (NL) and time-invariant (TI);
(C) It's linear (L) and time-varying (TV);
(D) It's nonlinear (NL) and time-varying (TV).

8. (8%) The figure below shows a trivial *quadrature mirror filter* (QMF) bank separating even-indexed and odd-indexed samples $x[2m]$ and $x[2m-1]$ into two channels and then recombined to form $x[n]$. The downsamplers, denoted by $\downarrow 2$, decrease the sampling rate by a factor of two in each channel by dropping every other sample, i.e., $v_1[m] = x[2m]$. The corresponding upsamplers, denoted by $\uparrow 2$, increase the sampling rate by a factor of two, on the other and, by inserting a sample of zero value between each pair of input samples, i.e., $w_1[n] = \begin{cases} v_1[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$. What is the relationship between the z-transform of $x[n]$ and $w_2[n]$?
(A) $W_2(z) = X(z^{-1})$;
(B) $W_2(z) = z^{-1}X(z)$;
(C) $W_2(z) = \frac{1}{2}(X(z) + X(-z))$;
(D) $W_2(z) = \frac{1}{2}(X(z) - X(-z))$;
(E) $W_2(z) = \frac{1}{2}z^{-1}(X(z) + X(-z))$



Part 2 : Please simplify your answers as much as possible and write down detailed steps

(9) Three equal-probability messages with indexes j are mapped into symbol waveforms as

$$u(t,j) = \begin{cases} \frac{-\sqrt{2}}{2} sinc(t) + \frac{3\sqrt{2}}{2} sinc(3t) & \text{if } j=1 \\ \left(\frac{\sqrt{2}}{4} - \frac{\sqrt{3}}{2}\right) sinc(t) - \frac{3\sqrt{2}}{4} sinc(3t) & \text{if } j=2 \\ \left(\frac{\sqrt{2}}{4} + \frac{\sqrt{3}}{2}\right) sinc(t) - \frac{3\sqrt{2}}{4} sinc(3t) & \text{if } j=3 \end{cases}$$

(a) (6%) Normalize the symbol waveform $u(t,1)$.
(b) (6%) If we denote your answer in (a) as $\phi_1(t)$, please find the projection of $u(t,2)$ on $\phi_1(t)$.
(c) (9%) Find a set of orthonormal bases and the corresponding coordinates of three $u(t,j)$.
(10) Consider a binary erasure channel (BEC) with transition probability $W(y|x)$, where the channel output y equals to the binary input x with $W(x|x) = 1-\epsilon$ or an erasure $*, 0 \le \epsilon \le 1$.
(a) (4%) We use $\oplus$ to denote the binary addition or the XOR operation. Now we have two bits u_1, u_2 and by encoding them as $x_1 = u_1 \oplus u_2, x_2 = u_2$ and pass (x_1, x_2) to i.i.d. BECs to get (y_1, y_2), a two-input two-output vector channel with transition probability

$$W_2(y_1, y_2|u_1, u_2) = W(y_1|u_1 \oplus u_2)W(y_2|u_2).$$

is formed. Please find $W_2(*, 0|1, 0)$ and $W_2(*, 1|1, 0)$.
(b) (5%) Based on channel W_2 we form a channel $W_2^{(1)}(\hat{u}_1|y_1, y_2)$ and please find the minimum error probability $\Pr(\hat{u}_1 \ne u_1|u_1, u_2)$ from all possible $W_2^{(1)}$.
(c) (5%) As (b), now we form another channel $W_2^{(2)}(\hat{u}_2|y_1, y_2, u_1)$ and please find the minimum error probability $\Pr(\hat{u}_2 \ne u_2|u_1, u_2)$ from all possible $W_2^{(2)}$
(d) (5%) Express your answers in (b) and (c) by functions of ϵ as $f^{(1)}(\epsilon)$ and $f^{(2)}(\epsilon)$ respectively. Now construct a discrete time random process $H_n, n = 1, 2, \cdots$ as follows

$$H_{n+1} = \begin{cases} f^{(1)}(H_n) & \text{with } 1/2 \text{ probability} \\ f^{(2)}(H_n) & \text{otherwise.} \end{cases}$$

where H_0 is fixed and equal to ϵ. Show that the conditional mean $\mathbb{E}[H_{n+1}|H_1, \dots, H_n] = H_n$.
(e) (10%) Assume when $n \to \infty, H_\infty$ is finite. Find all possible values of H_∞ with explanation.

試題隨卷繳回