- Any device with a computer algebra system is prohibited during the exam.
- There are FIVE questions in total. Label the question numbers **clearly** on your work.
- Answer all questions. You will have to **show all of your calculations or reasoning** to obtain credits.

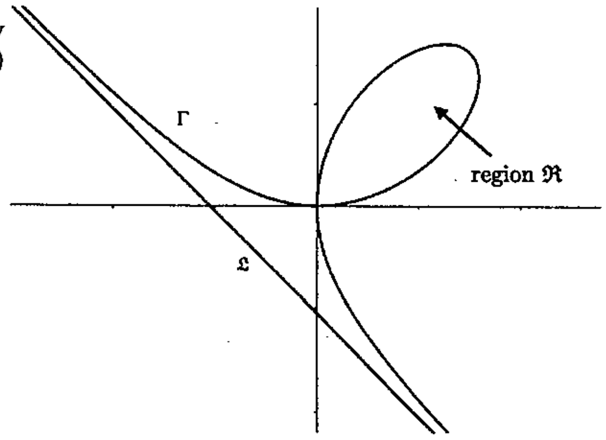
1. (20%)

   (a) Find the first derivative of $f(\theta) = \dfrac{1}{1+\tan^3\theta}$.

   (b) Let $a > 0$ be a real number. Consider the curve $\Gamma : x^3 + y^3 = 3axy$.

   (i) The equation of Γ defines implicitly $y = y(x)$ for x sufficiently large. Find the equation of the slant asymptote $\mathfrak{L}$ of $y = y(x)$ (when $x \to \infty$).

   (ii) Find the area enclosed by the loop of Γ (the region $\mathfrak{R}$).



2. (20%) Let a be a real number such that $|a| < 1$.

   (a) (i) Let n be a positive integer. Compute $\int_0^\infty x^n e^{-x}\,dx$.

   (ii) Write down the Maclaurin series (Taylor series at $x = 0$) of $1 - e^{-ax}$.

   (b) In this part, you may assume, without proof, that the given improper integrals are convergent.

   (i) Use (a) to find the exact value of $\displaystyle\int_0^\infty \frac{(1-e^{-ax})e^{-x}}{x}\,dx - \ln(1+a)$.

   (ii) Hence, evaluate the integral $\displaystyle\int_0^1 \frac{x^p - x^q}{\ln x}\,dx$ where $|p| < 1$ and $|q| < 1$.

3. (20%) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(x,y) = \begin{cases} (x^2y)^{1/3}\cdot\sin\left(\dfrac{y}{x}\right) & \text{if } x \neq 0; \\ 0 & \text{if } x = 0. \end{cases}$

   (a) For $a \neq 0$, find the partial derivatives $\dfrac{\partial f}{\partial x}(a,0)$ and $\dfrac{\partial f}{\partial y}(a,0)$.

   (b) Find the directional derivative of f at $(0,0)$ in the direction $\vec{u} = \left\langle \dfrac{3}{5}, -\dfrac{4}{5}\right\rangle$.

   (c) Determine <u>all</u> the points of $\mathbb{R}^2$ at which f is differentiable. Justify your answer.

4. (20%) A space curve C is formed by intersecting the ellipsoid $\mathfrak{E} : x^2 + 4y^2 + z^2 = 4$ and the cylinder $\mathfrak{L} : (x-1)^2 + z^2 = 1$.

   (a) Find the absolute maximum and absolute minimum values (if exists) of $f(x,y,z) = x^2 + y^2$ on the curve C by the method of Lagrange's multipliers.

   (b) Is it possible to find a point on C at which the osculating plane is parallel to the xz-plane ? Explain.

5. (20%) Evaluate the following double integrals.

   (a) $\displaystyle\int_{\pi/4}^{3\pi/4}\int_0^{\frac{6}{2\sin\theta-\cos\theta}} r^2\cos\theta\,dr\,d\theta$.

   (b) $\displaystyle\int_1^2\int_{\frac{1}{y}}^{y} \sqrt{\frac{y}{x}}\cdot e^{\sqrt{xy}}\,dx\,dy$.

試題隨卷繳回